

Geospatial statistics and spatial data interpolation methods



ERASMUS Intensive Program GIS'EM 2013 at Eberswalde

Prof. Dr. Jan-Peter Mund

University for Sustainable Development, Eberswalde

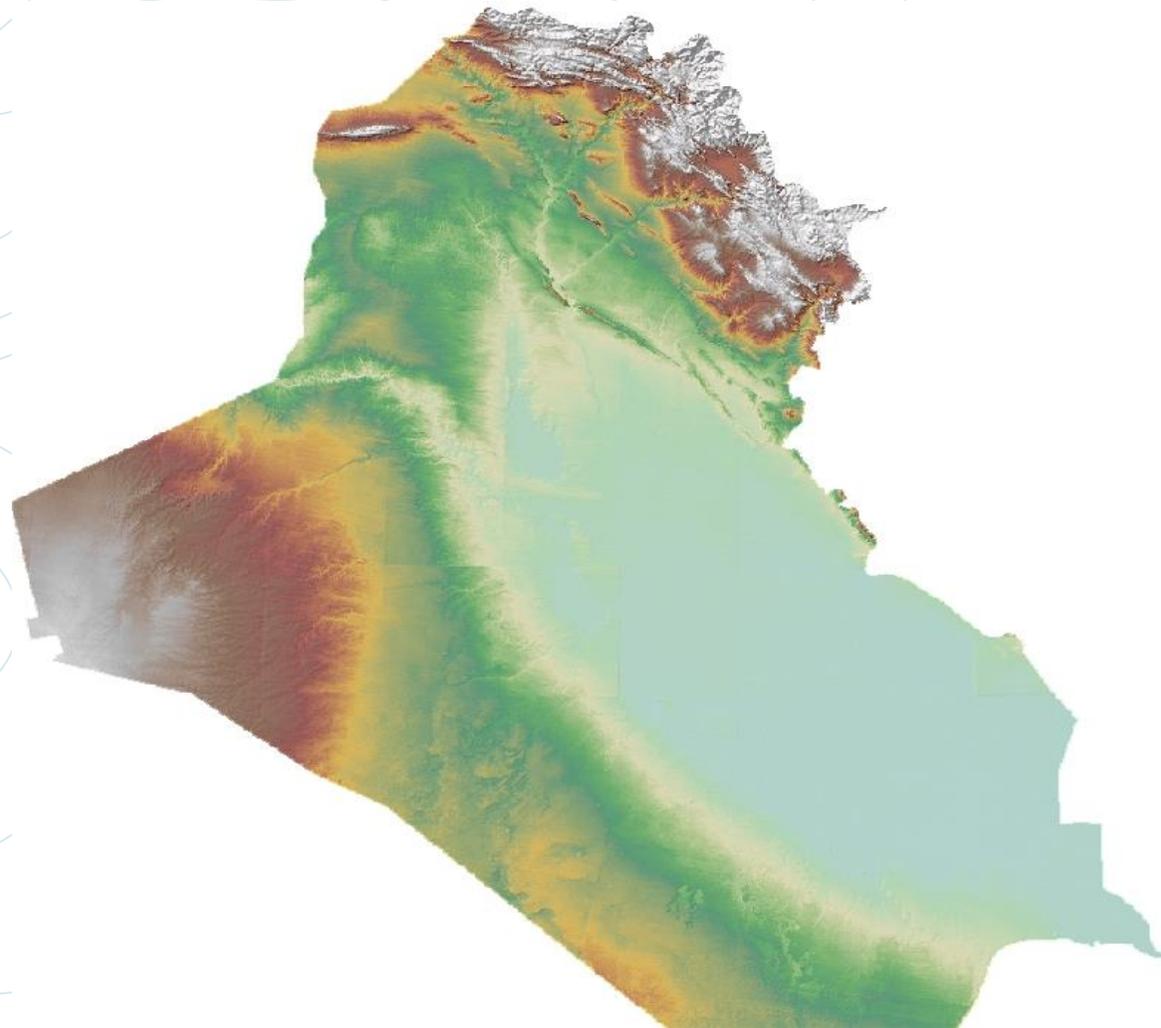
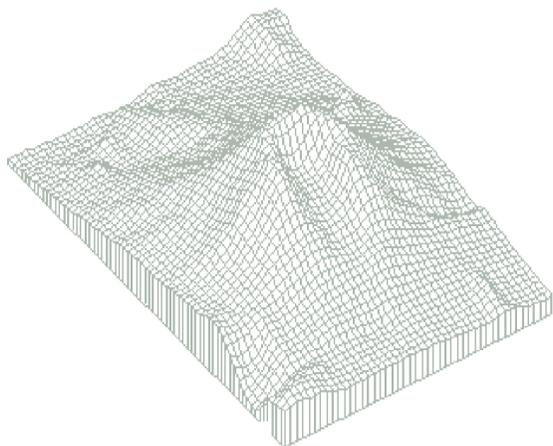
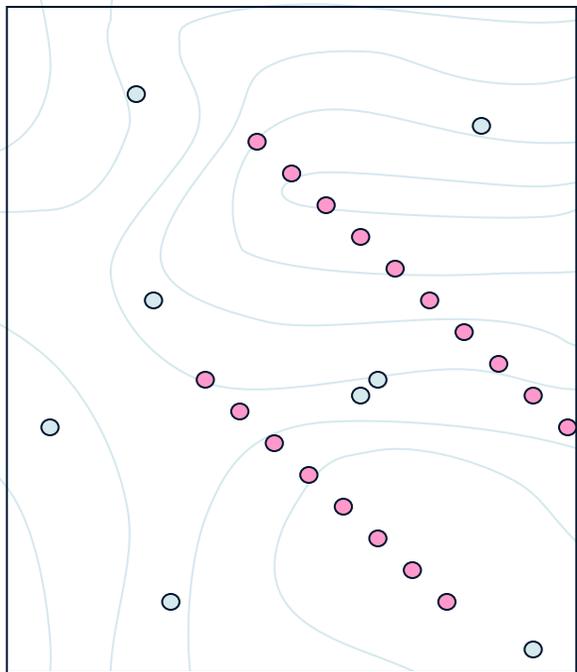


GIS'EM - Intensive Programme on GIS in Environmental Management

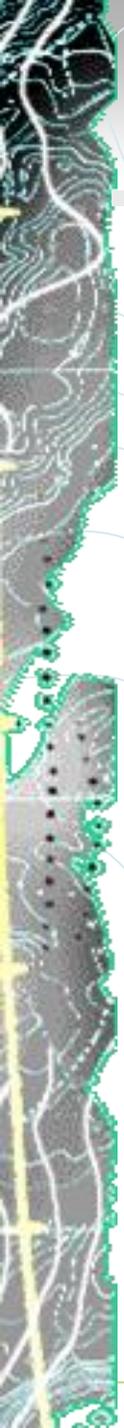
4th March – 15th March 2013,
Eberswalde Germany



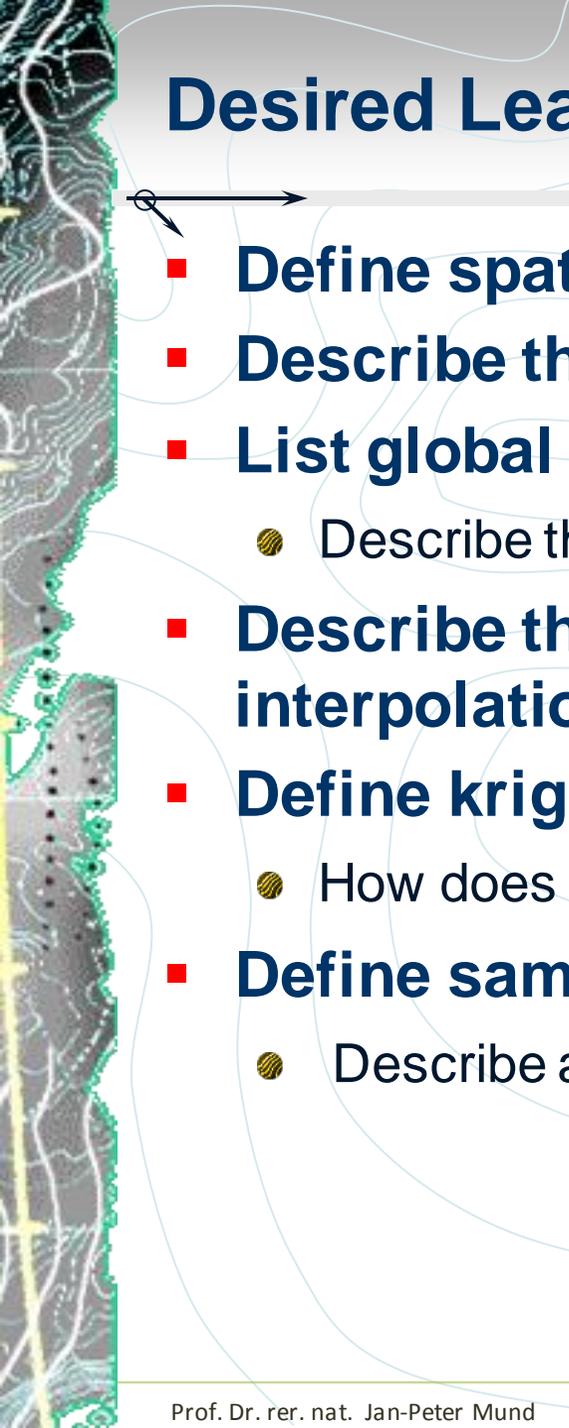
Spatial data interpolation



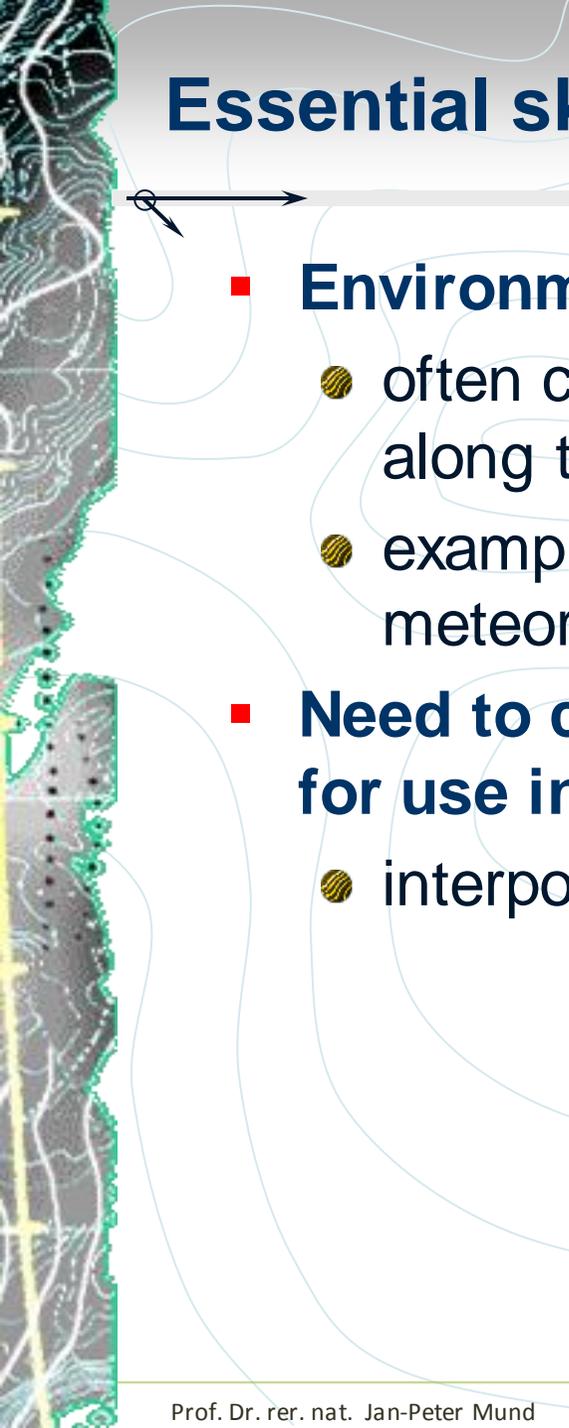
Outline - Exploring Continuous Data

- 
- **Spatial Interpolation**
 - **Sampling**
 - **Triangulation**
 - **Inverse Distance Weighting (IDW)**
 - **Trend**
 - **Spline**
 - **Kriging**

Desired Learning Objectives

- 
- **Define spatial interpolation**
 - **Describe the inputs needed for spatial interpolation**
 - **List global and local interpolation methods;**
 - Describe the differences between the 2 methods
 - **Describe the difference between an exact and an inexact interpolation method**
 - **Define kriging**
 - How does it differ from other interpolation methods?
 - **Define sampling methodology**
 - Describe applications of various sampling methodologies

Essential skill analysing environmental data



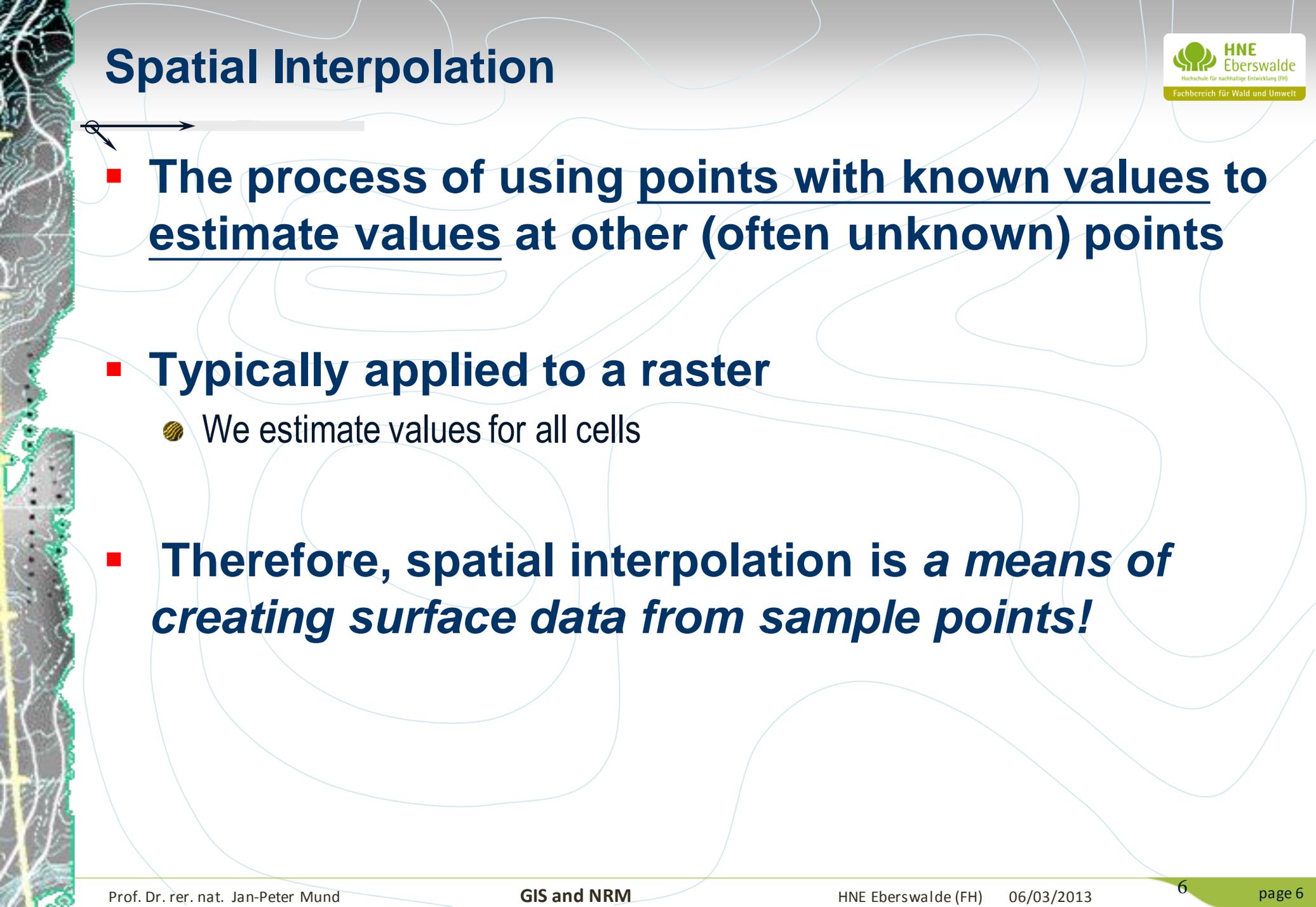
- **Environmental data**

- often collected as discrete observations at points or along transects
- example: soil cores, soil moisture, vegetation transects, meteorological station data, etc.

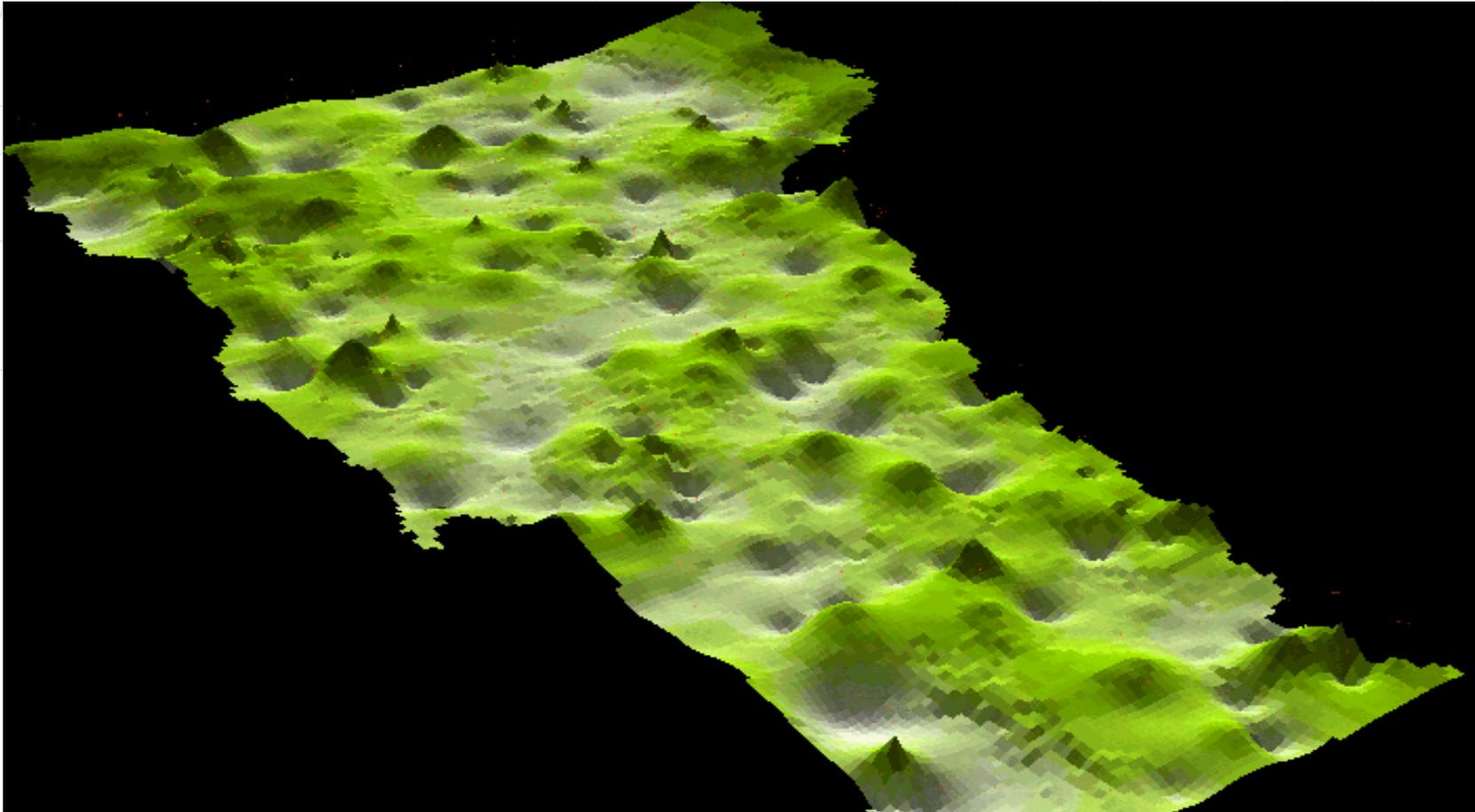
- **Need to convert discrete data into continuous surface for use in GIS modelling**

- interpolation

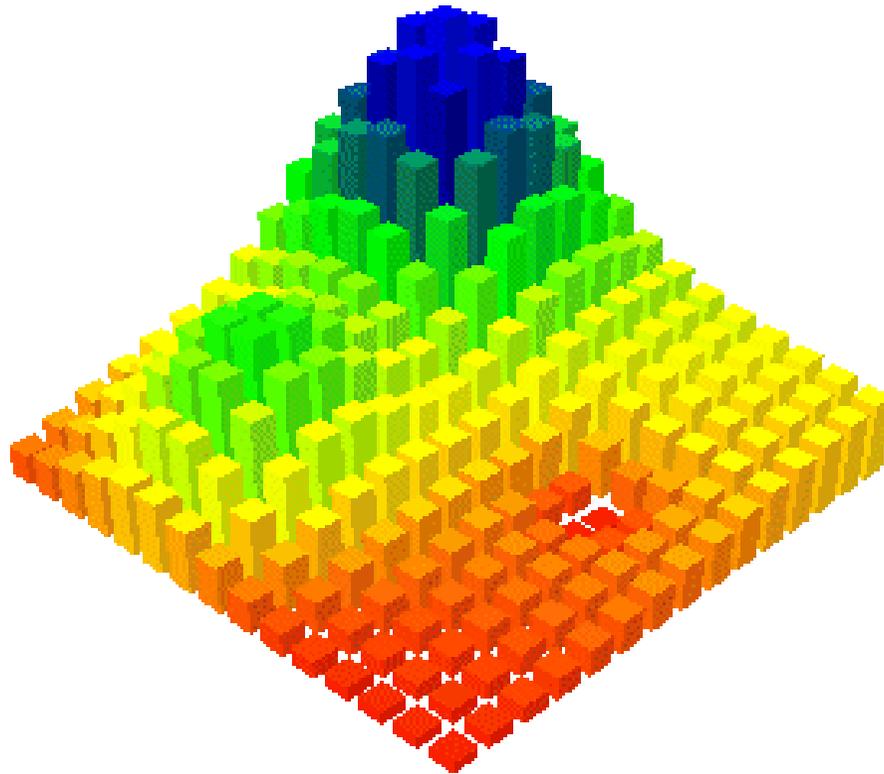
Spatial Interpolation

- 
- ➔
- **The process of using points with known values to estimate values at other (often unknown) points**
 - **Typically applied to a raster**
 - We estimate values for all cells
 - **Therefore, spatial interpolation is *a means of creating surface data from sample points!***

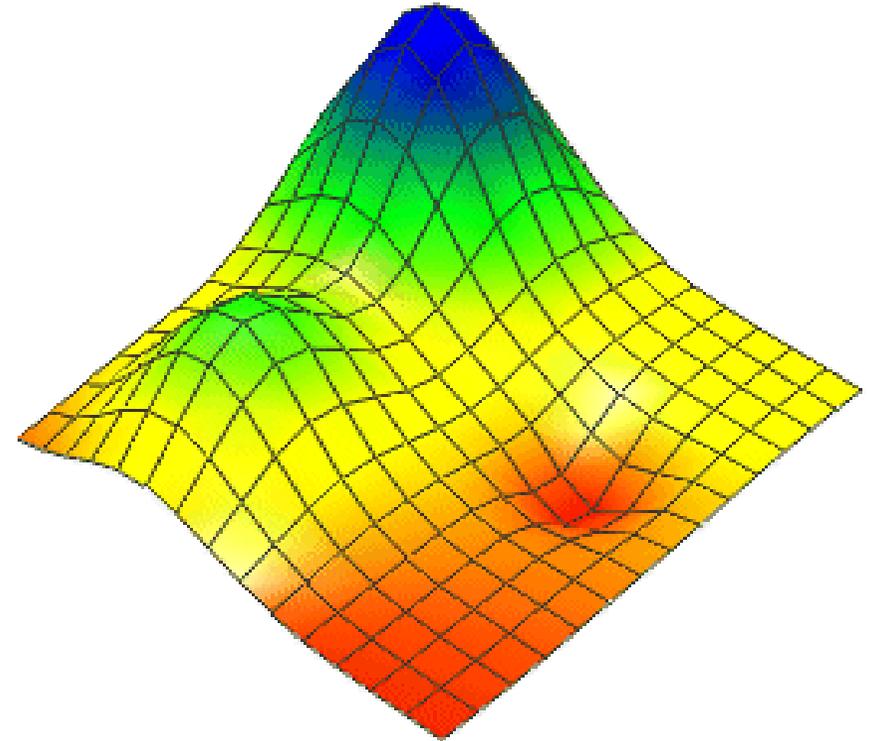
How is it used...



Which One's the DEM?



(a) Discrete Elevation Samples



(b) Implicit (Linear) Continuous Surface



■ Definition:

“Spatial interpolation is the procedure of estimating the values of properties at unsampled sites within an area covered by existing observations.”
(Waters, 1989)

■ Complex problem

- wide range of applications
- important in addressing problem of data availability
- quick fix for partial data coverage
- interpolation of point data to surface/polygon data
- role of filling in the gaps between observations

What is spatial analysis?



- **Methods for working with spatial data**

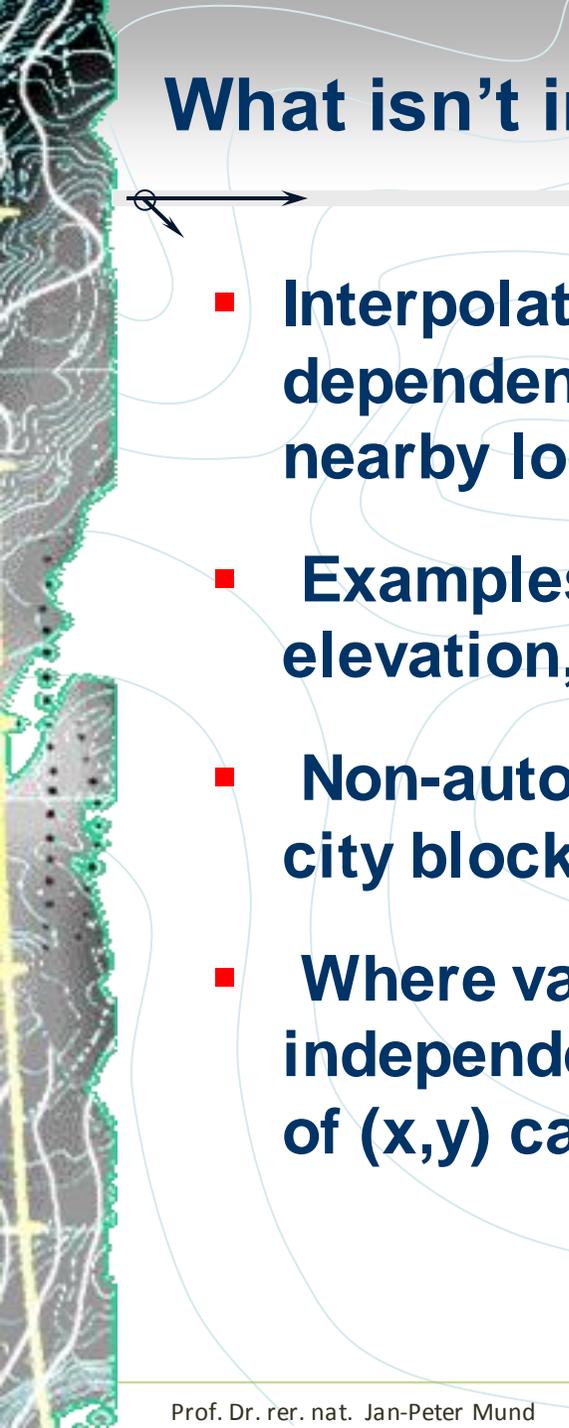
- to detect patterns, anomalies
- to find answers to questions
- to test or confirm theories
 - deductive reasoning
- to generate new theories and generalizations
 - inductive reasoning

- **"a set of methods whose results change when the locations of the objects being analyzed change"**

What is interpolation?

- 
- **Process of creating a surface based on values at isolated sample points.**
 - **Sample points are locations where we collect data on some phenomenon and record the spatial coordinates**
 - **We use mathematical estimation to “guess at” what the values are “in between” those points**
 - **We can create either a raster or vector interpolated surface**
 - **Interpolation is used because field data are expensive to collect, and can't be collected everywhere**

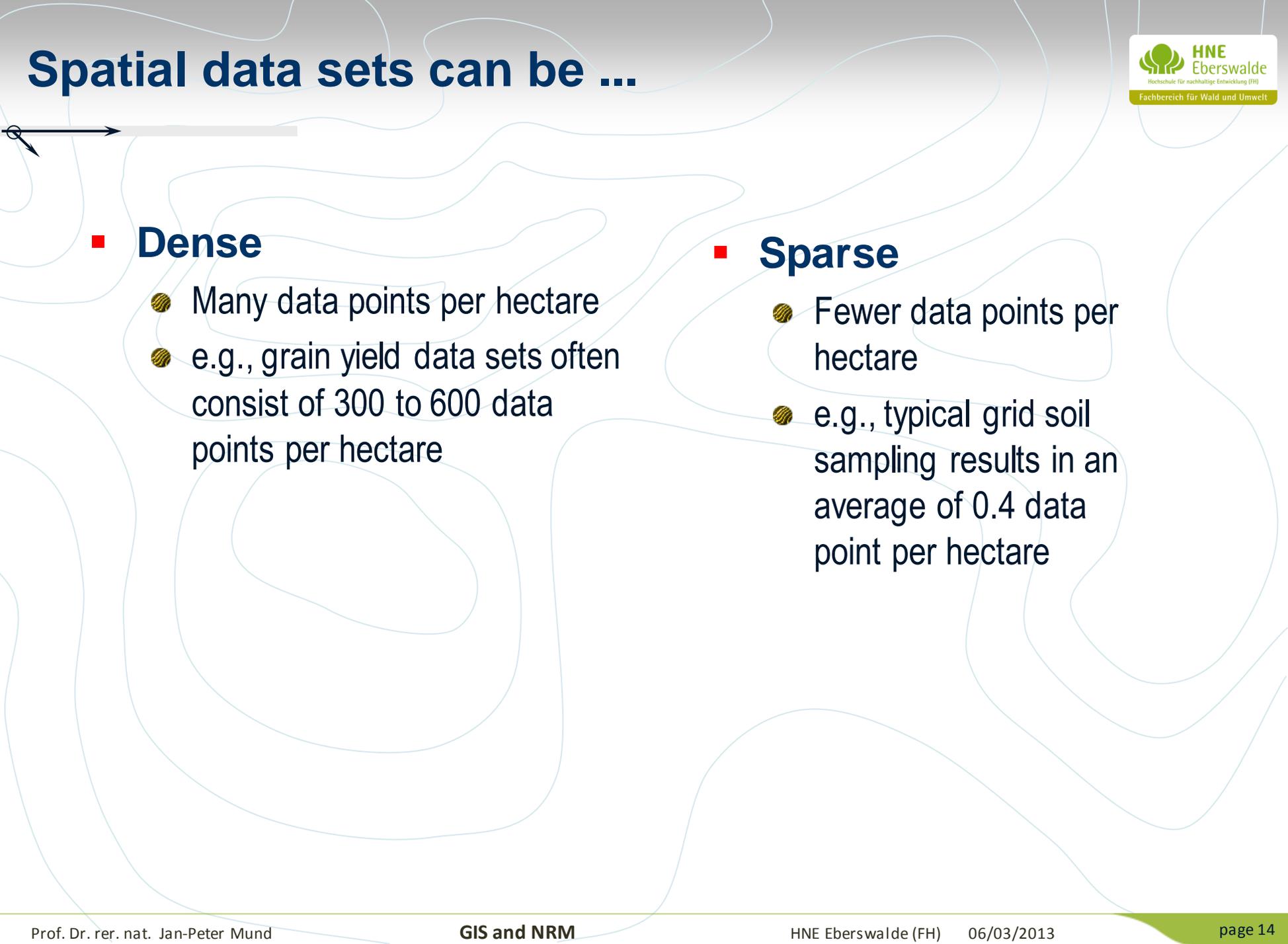
What isn't interpolation?

- 
- **Interpolation only works where values are spatially dependent, or spatially autocorrelated, that is, where nearby locations tend to have similar Z values.**
 - **Examples of spatially autocorrelated features: elevation, property value, crime levels, precipitation**
 - **Non-autocorrelated examples: number of TV sets per city block; cheeseburgers consumed per household.**
 - **Where values across a landscape are geographically independent, interpolation does not work because the value of (x,y) cannot be used to predict the value of (x+1, y+1).**

Where interpolation does NOT work

- Cannot use interpolation where values are not spatially autocorrelated
- Say looking at household income—in an income-segregated city, you could take a small sample of households for income and probably interpolate
- However, in a highly income-integrated city, where a given block has rich and poor, this would not work

Spatial data sets can be ...



- **Dense**

- Many data points per hectare
- e.g., grain yield data sets often consist of 300 to 600 data points per hectare

- **Sparse**

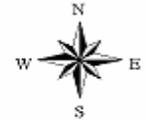
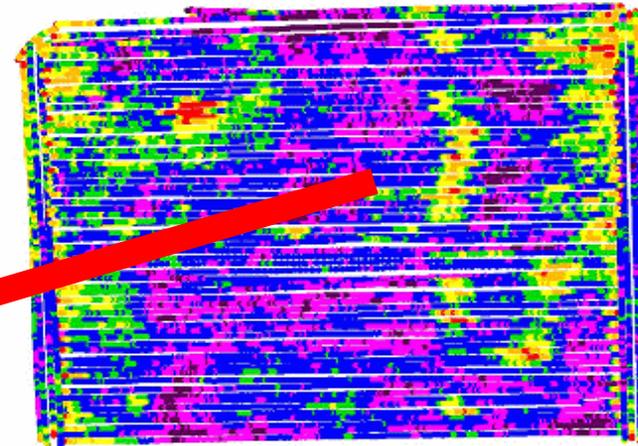
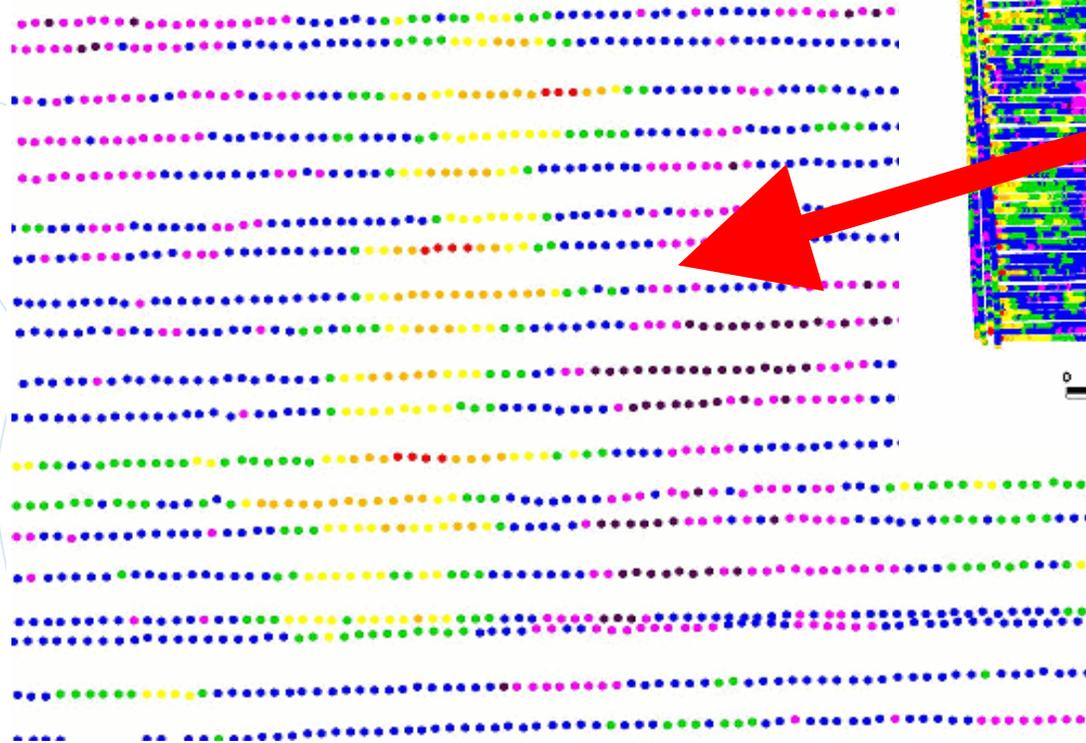
- Fewer data points per hectare
- e.g., typical grid soil sampling results in an average of 0.4 data point per hectare

Yield data are dense ...

■ One sec. readings at 1-100 data point every meter

- 600-3000 data points per hectare with a 6-row combine header

Yield98



M1-1998-corn-all.shp

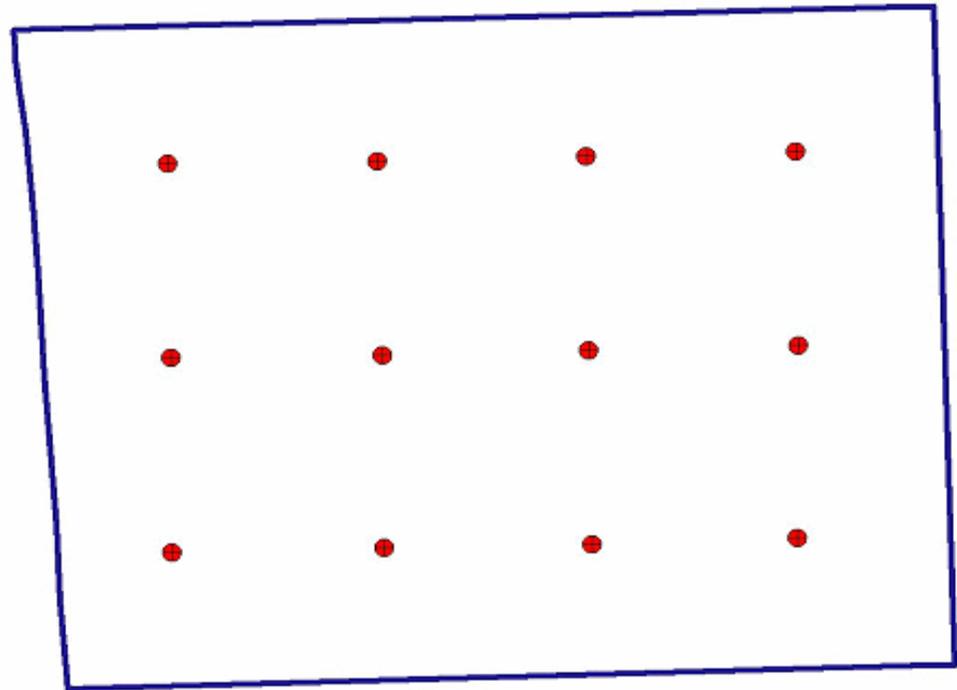
- 0 - 25
- 25 - 50
- 50 - 75
- 75 - 100
- 100 - 125
- 125 - 150
- 150 - 238

0 400 800 Feet

Soil sample data are sparse

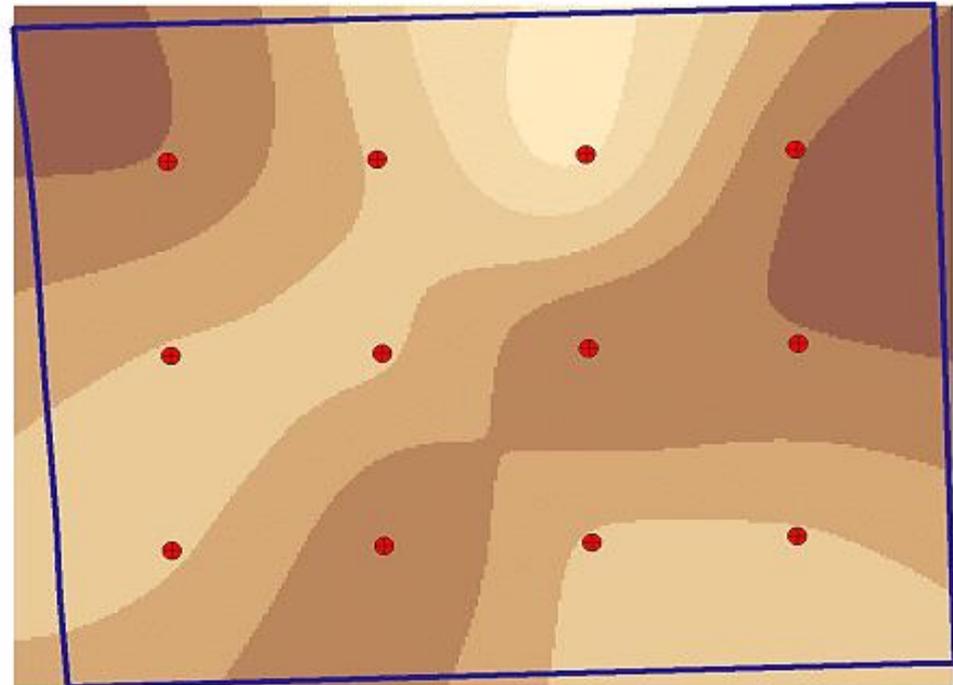
■ Typical 1 hectare sampling grid

- Only 6-12 point per hectare



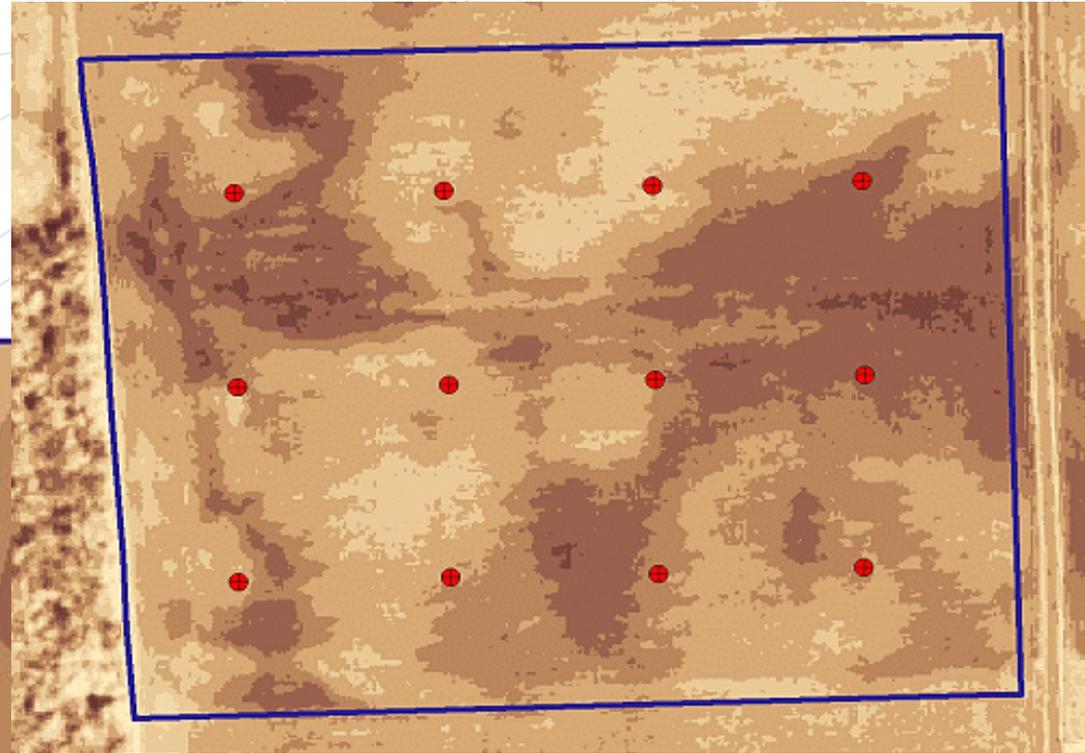
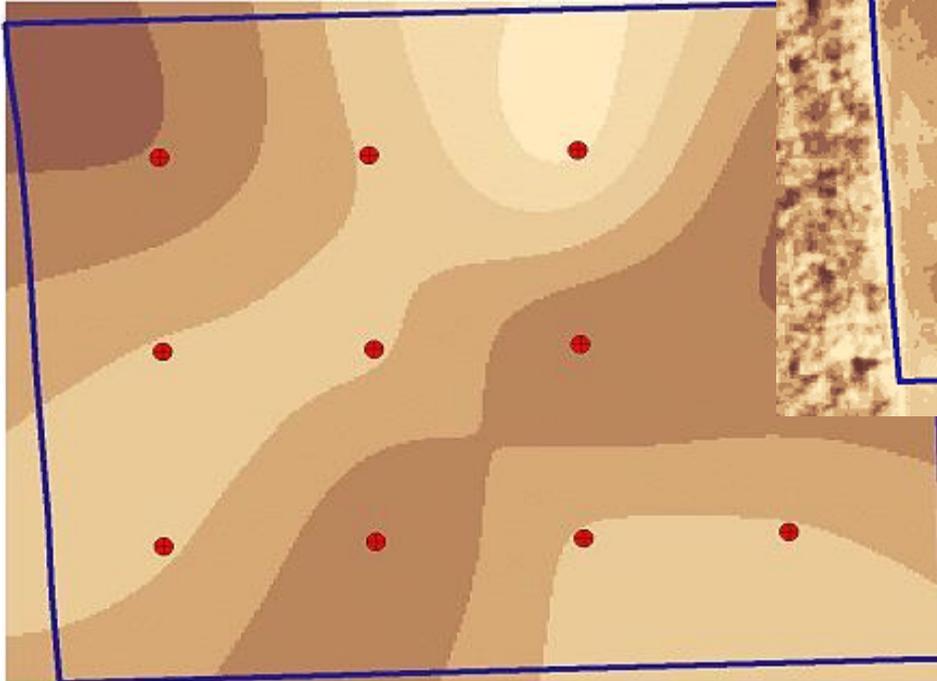
Organic matter surface map

- Interpolated from values of 1 ha soil sample data



Reality check

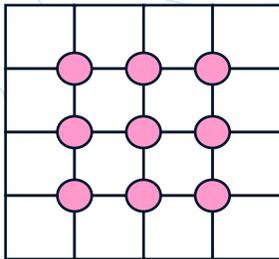
- Soil surface color from reclassified aerial IR



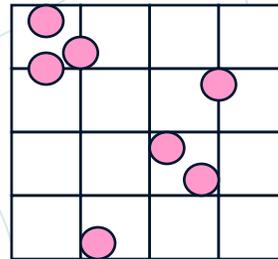
- Soil surface map interpolated from 1 ha samples

Data sampling

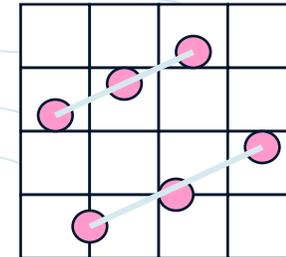
- Method of sampling is critical for subsequent interpolation...



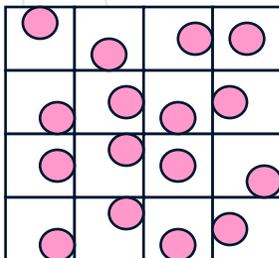
Regular



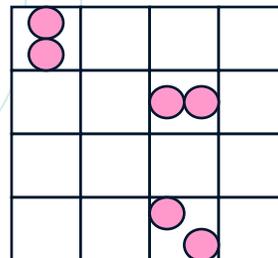
Random



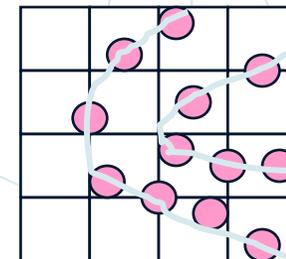
Transect



Stratified random



Cluster



Contour

■ Systematic sampling pattern

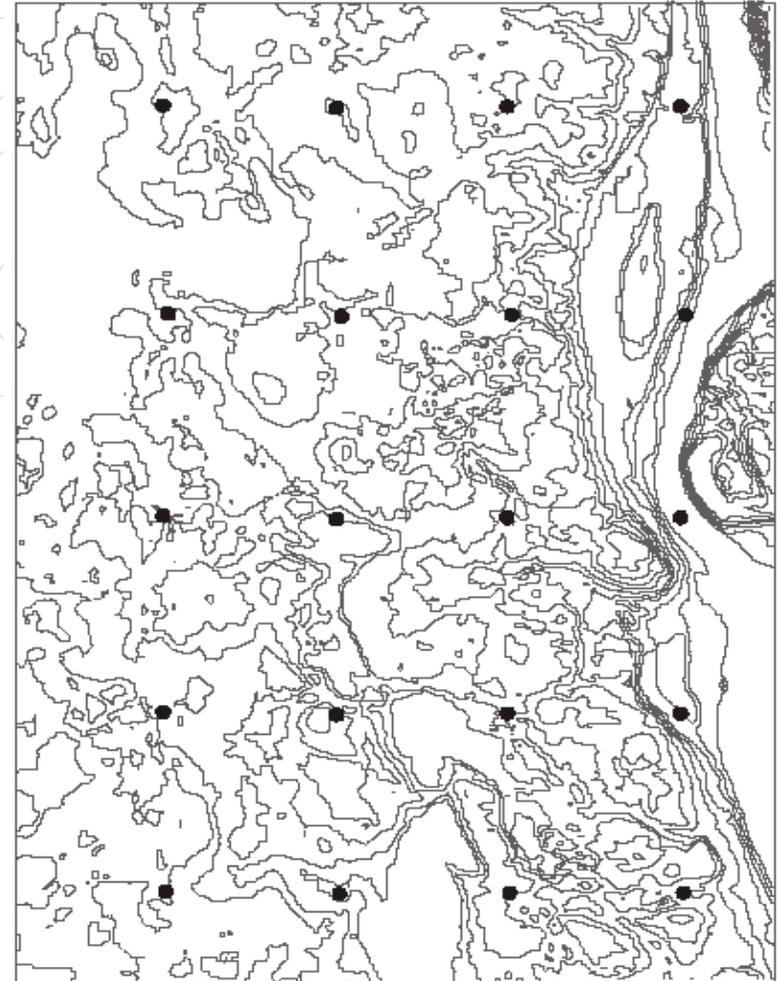
- Easy
- Samples spaced uniformly at fixed X, Y intervals
- Parallel lines

■ Advantages

- Easy to understand

■ Disadvantages

- All receive same attention
- Difficult to stay on lines
- May be biases



Sampling

■ Random Sampling

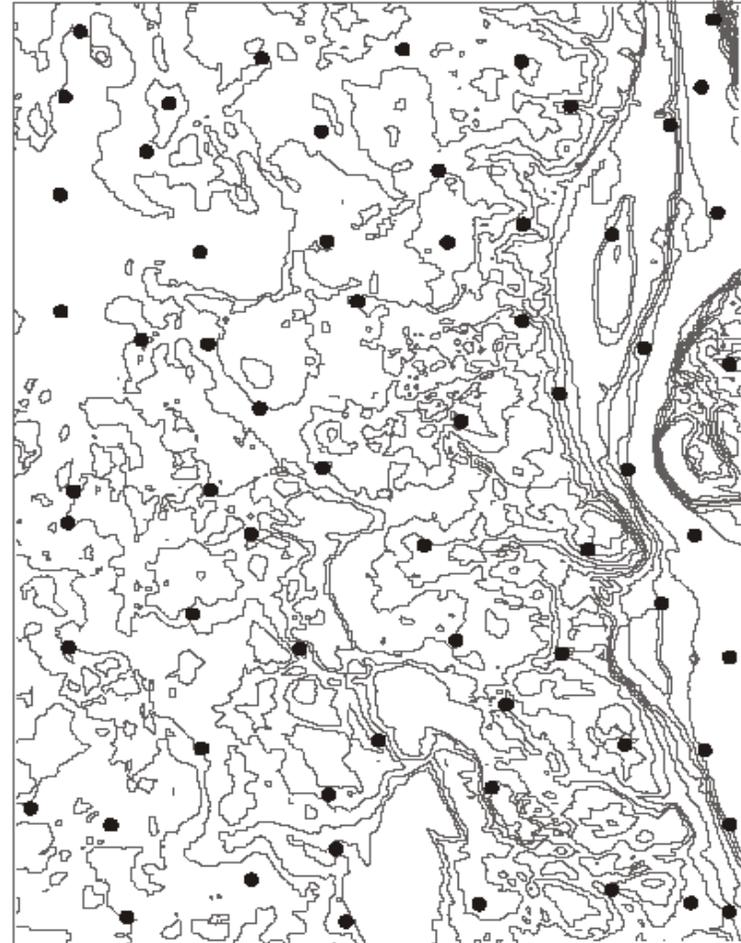
- Select point based on random number process
- Plot on map
- Visit sample

■ Advantages

- Less biased (*unlikely to match pattern in landscape*)

■ Disadvantages

- Does nothing to distribute samples in areas of high
- Difficult to explain, location of points may be a problem



Cluster Sampling

- Cluster centers are established (*random or systematic*)
- Samples arranged around each center
- Plot on map
- Visit sample
 - (e.g. Forest Services, Forest Inventory Analysis (FIA))
 - Clusters located at random then systematic pattern of samples at that location)

Advantages

- Reduced travel time



■ Adaptive sampling

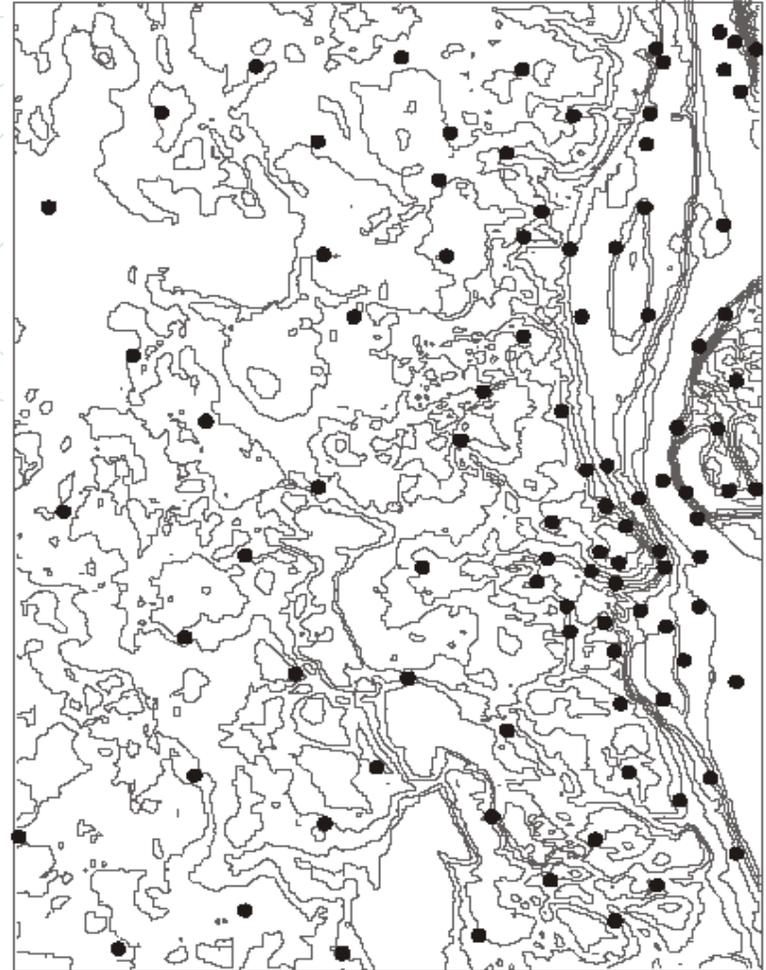
- Higher density sampling where the feature of interest is more variable.
- Requires some method of estimating feature variation
- Often repeat visits (e.g. two stage sampling)

■ Advantages

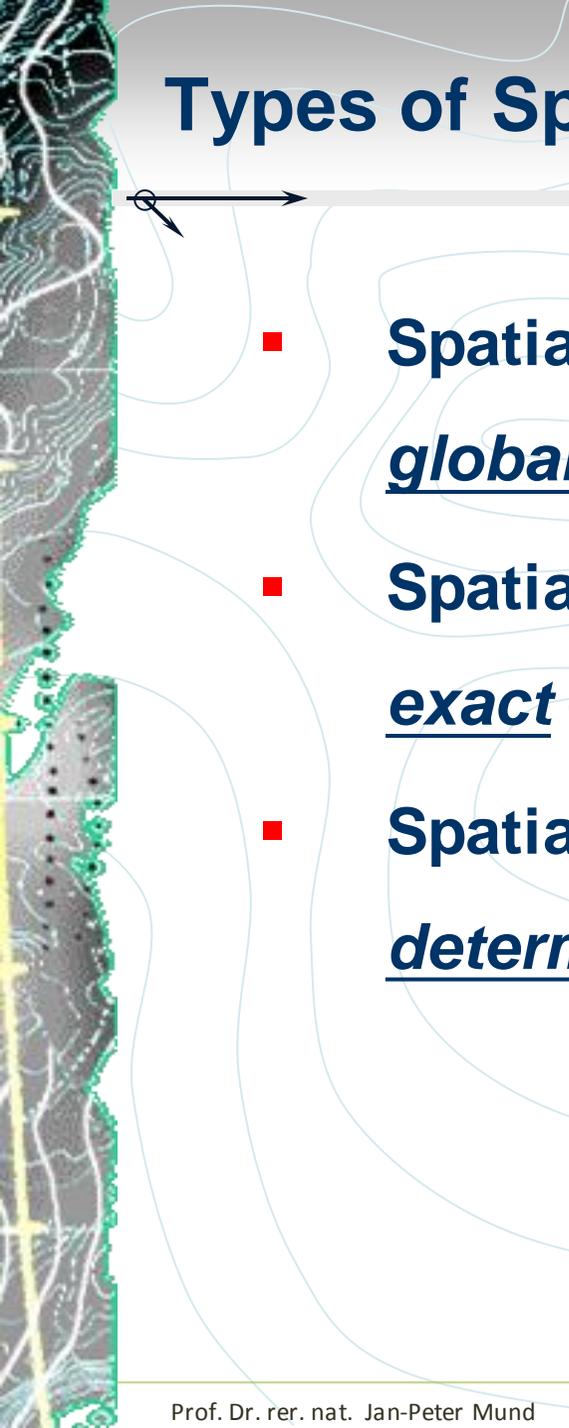
- Often efficient as large homogeneous areas have few samples reserving more for areas with higher spatial variation.

■ Disadvantages

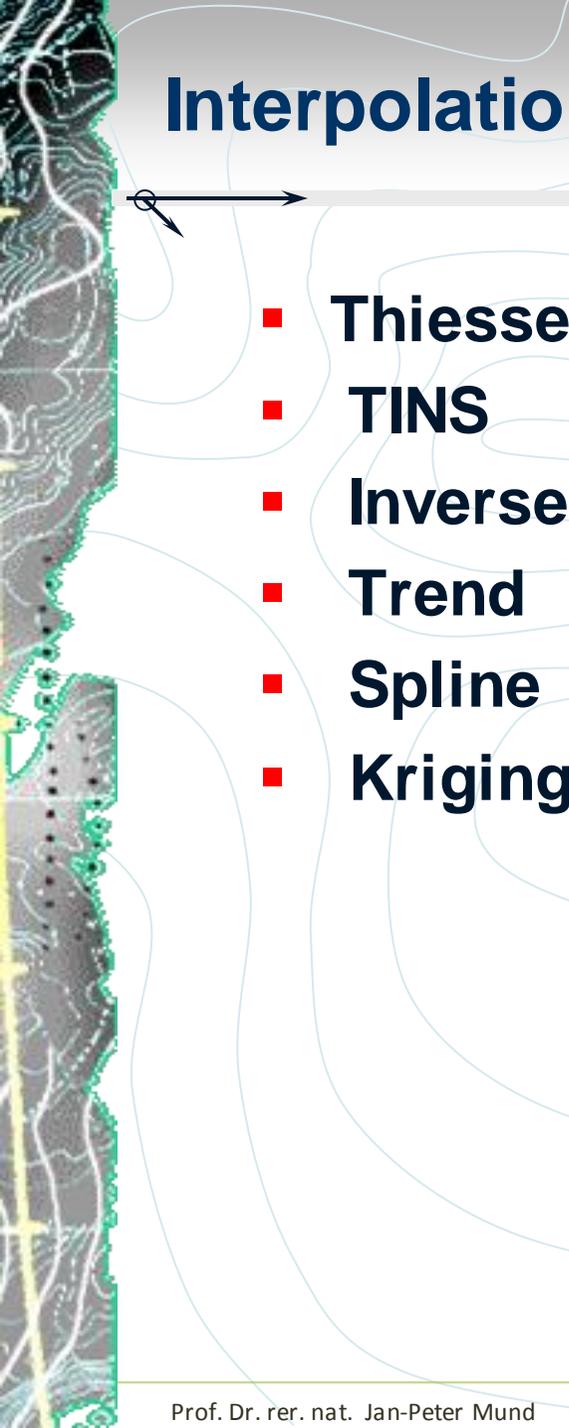
- If no method of identifying where features are most variable then several you need to make several sampling visits.



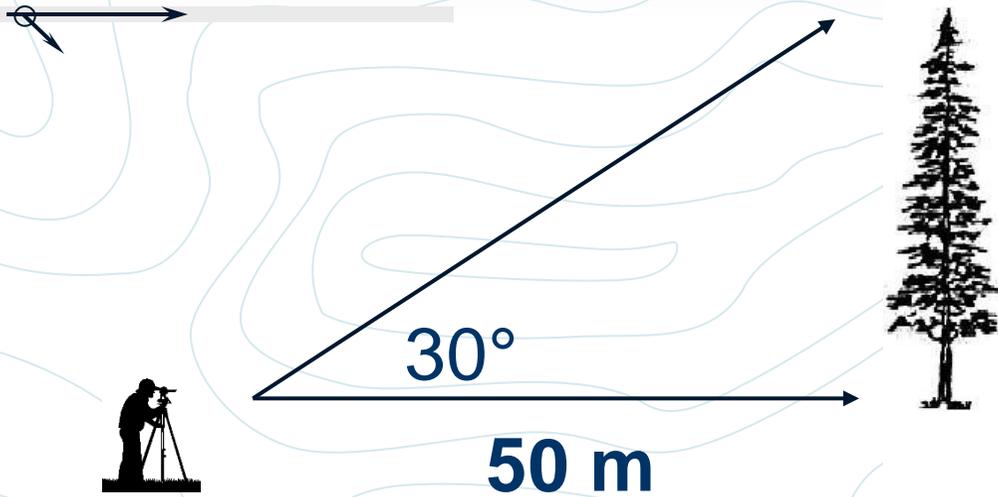
Types of Spatial Interpolation

- 
- Spatial interpolation can be global or local
 - Spatial interpolation can be exact or inexact
 - Spatial interpolation can be deterministic or stochastic

Interpolation Techniques

- 
- **Thiessen Polygons**
 - **TINS**
 - **Inverse Distance Weighting (IDW)**
 - **Trend**
 - **Spline**
 - **Kriging**

Simple Triangulation



$$\begin{aligned} \text{Tree height} &= \\ \tan (30^\circ) &* 50 \text{ m} \\ &= 28.86 \text{ m} \end{aligned}$$

Angle (degrees) = 30°

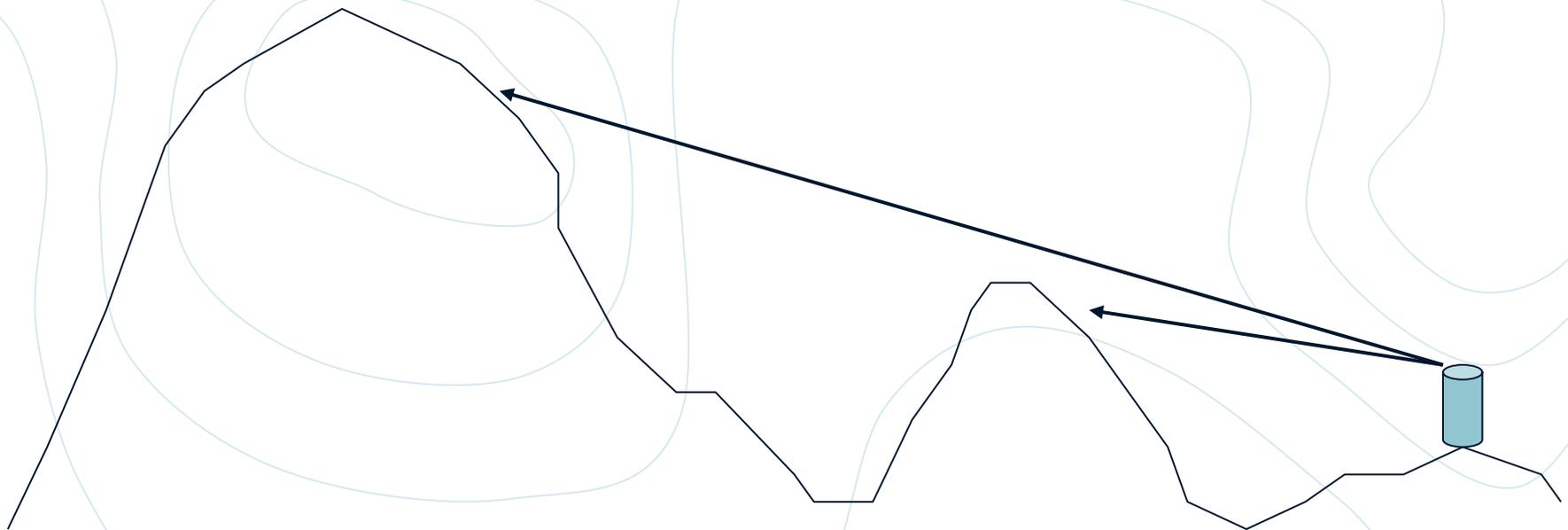
Angle (percent) = $(28.86 \text{ m} / 50 \text{ m}) = 57.7\%$

$\tan (30^\circ) 100 = 57.7$, providing a quick conversion from degrees to percent slope

A simple example of the conversion process from degrees to percent slope.

Visibility process

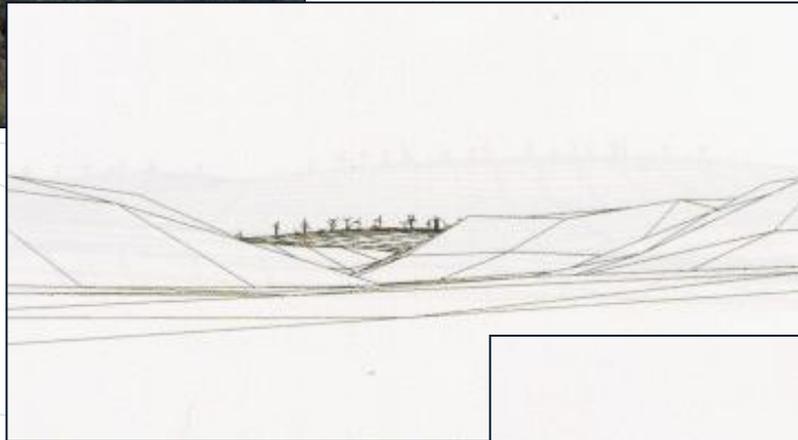
- Viewpoints (observers) are often point locations
- Operator can set limits (distance, angle, height)
- Line of sight from a viewing site to the surrounding landscape



Wind farm – photomontage



before



after

**wind –farm
CAD frame
model**



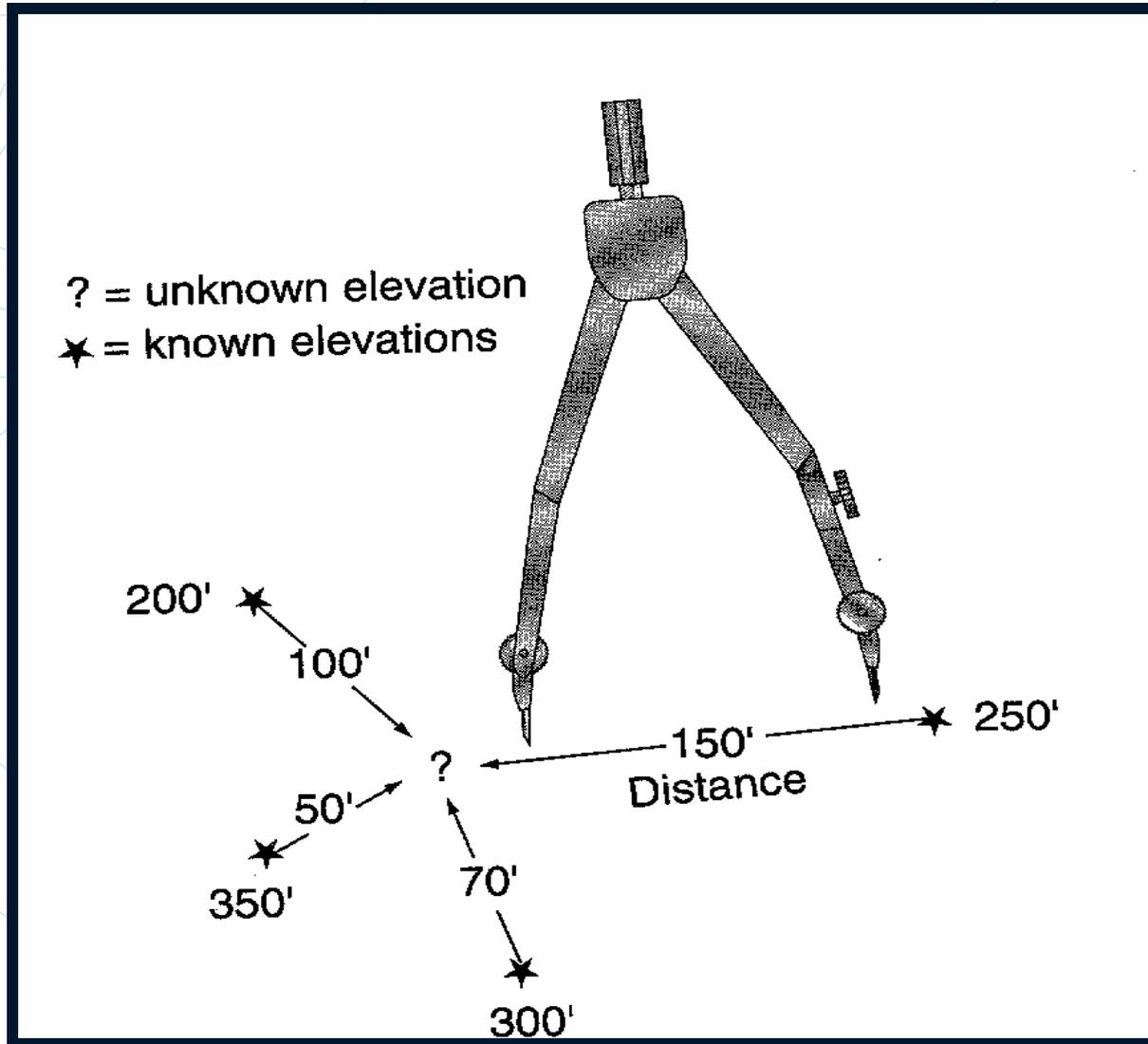
Spatial data interpolation – Principles

Concepts and principles



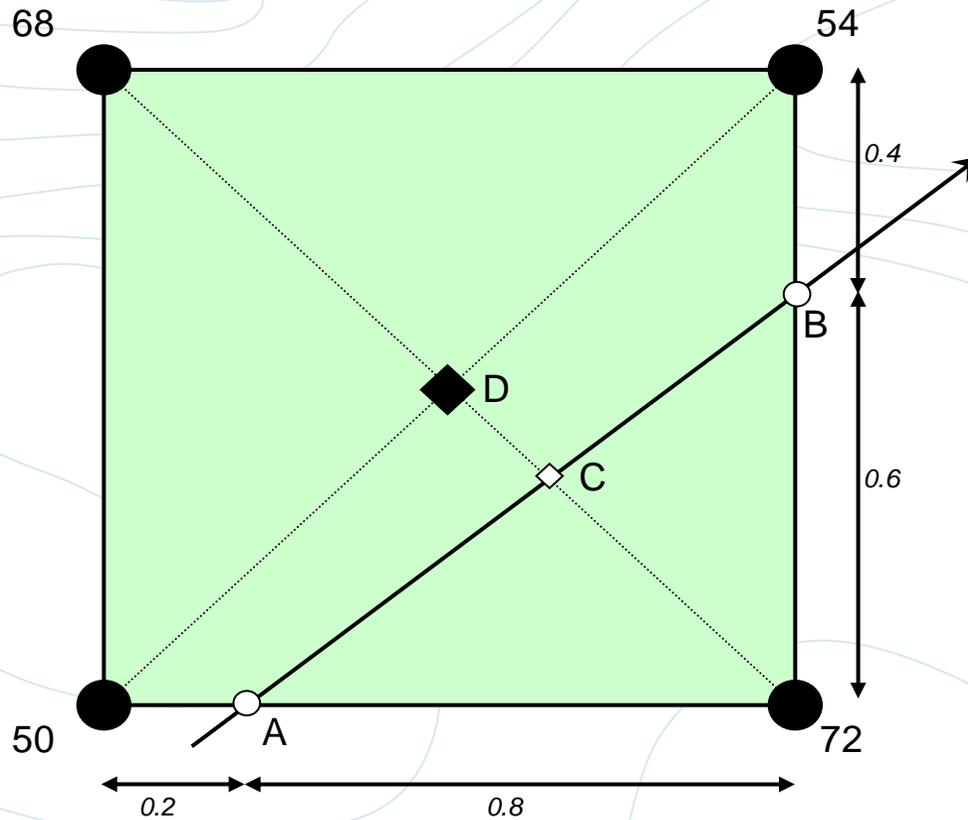
456	405	621	555	574	571	550	526	505	404	462	442	423	403	382	357	327	296	268	248	235
501	529	545	596	610	590	570	550	530	518	496	474	454	434	413	389	361	330	298	269	246
592	577	606	626	624	616	590	561	543	540	530	509	489	467	445	422	397	368	336	301	268
500	606	623	625	615	601	589	579	571	563	553	536	521	501	478	455	430	401	366	329	292
579	604	615	609	592	573	559	550	540	546	544	530	531	518	500	479	451	417	380	342	304
570	596	602	592	575	551	528	514	506	505	509	507	506	503	494	477	453	420	383	346	309
570	597	599	546	564	538	509	484	467	450	450	450	450	461	450	453	441	420	382	359	322
605	616	607	584	556	525	495	464	434	416	406	406	405	406	407	409	410	407	395	371	342
631	629	611	580	545	511	480	448	415	386	367	361	356	353	352	355	362	370	373	366	349
630	626	603	570	529	491	459	430	396	367	343	331	323	316	312	312	317	326	331	332	334
609	603	587	556	518	479	442	413	383	354	330	317	305	296	290	288	288	288	289	282	285
572	574	572	555	525	486	446	400	376	347	324	310	298	288	281	276	276	272	271	268	260
538	542	536	556	537	504	462	416	377	345	321	306	294	284	277	272	267	263	260	256	250
491	510	531	543	534	507	466	420	377	342	319	304	291	282	274	269	264	260	256	252	245
457	475	485	509	507	486	452	411	371	339	310	307	295	286	278	271	265	261	257	250	242
424	439	455	466	466	452	427	395	364	342	328	318	312	303	293	284	276	268	260	250	240
394	405	418	427	427	417	398	375	355	343	344	344	340	328	319	311	299	284	266	251	240
370	377	388	396	396	385	371	355	340	336	345	353	347	347	351	350	333	303	275	258	242
351	359	365	372	370	360	349	337	327	324	335	334	330	350	367	375	358	328	293	265	246
336	338	346	352	349	342	333	323	316	312	313	312	317	331	346	360	358	336	306	276	254
322	323	327	330	330	327	321	315	308	302	297	294	296	304	313	320	321	313	295	275	258

Spatial data interpolation – Principles

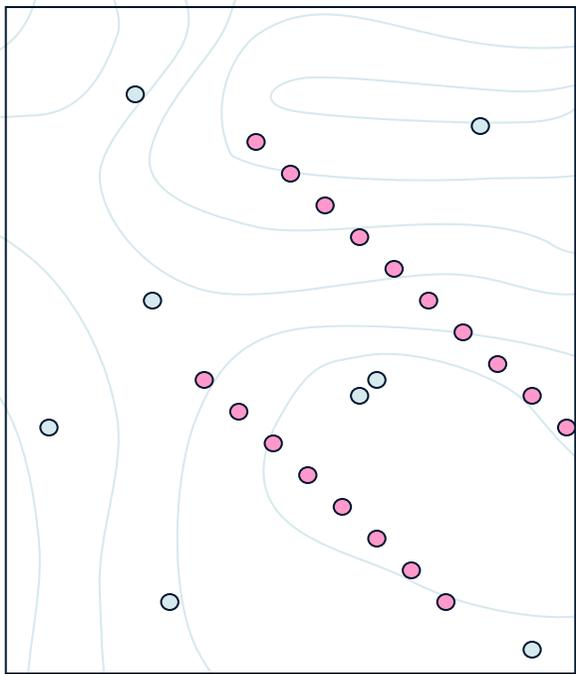


Interpolation for Visibility Analysis

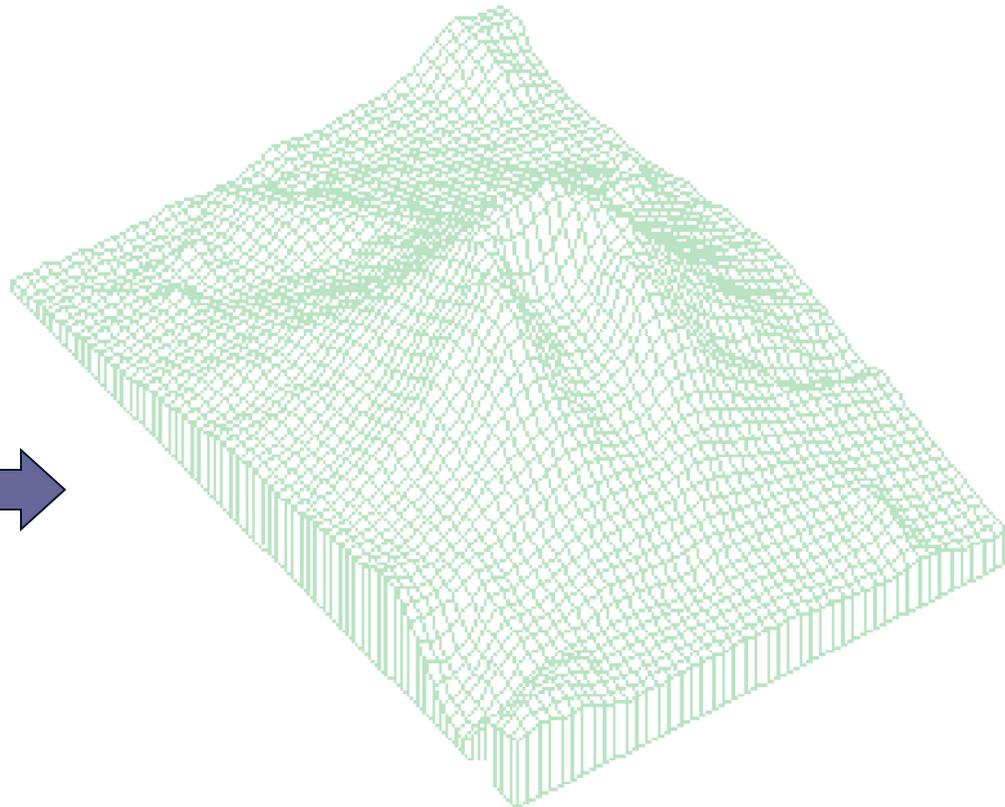
- What's the profile through the "cell" ?



Surfaces from points



Points

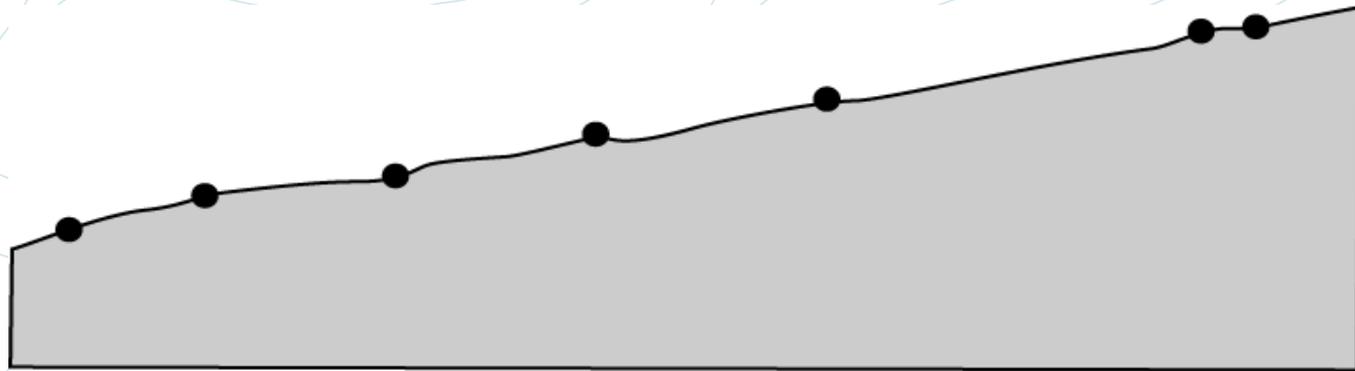


Surface

Control Points

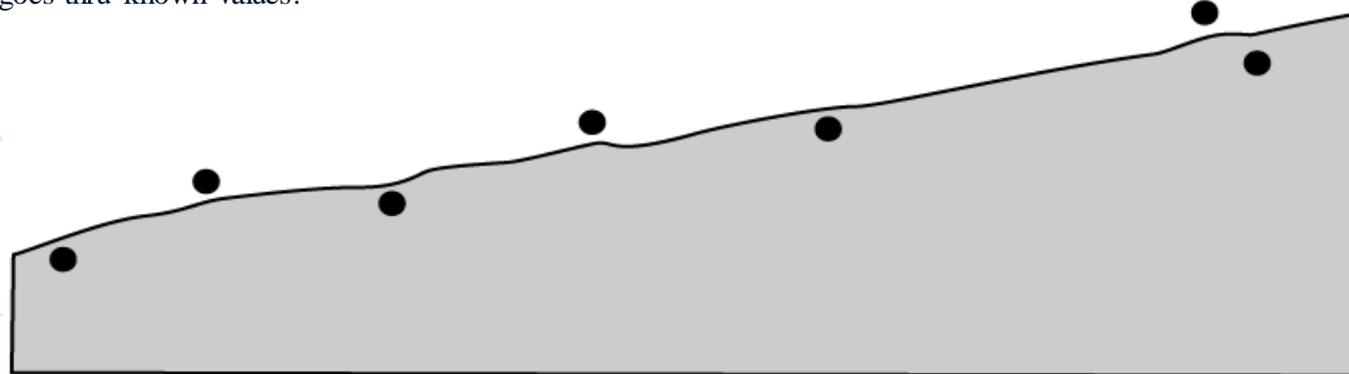
- **Control Points (CPs) are points with known values**
 - CPs provide the data necessary for the development of an *interpolator* for spatial interpolation
- **The *number* and *distribution* of control points can greatly influence the *accuracy* of spatial interpolation.**

Exact vs Inexact Interpolation



(a)

Exact interpolation goes thru known values!



(b)

DEM production with a long historical tradition



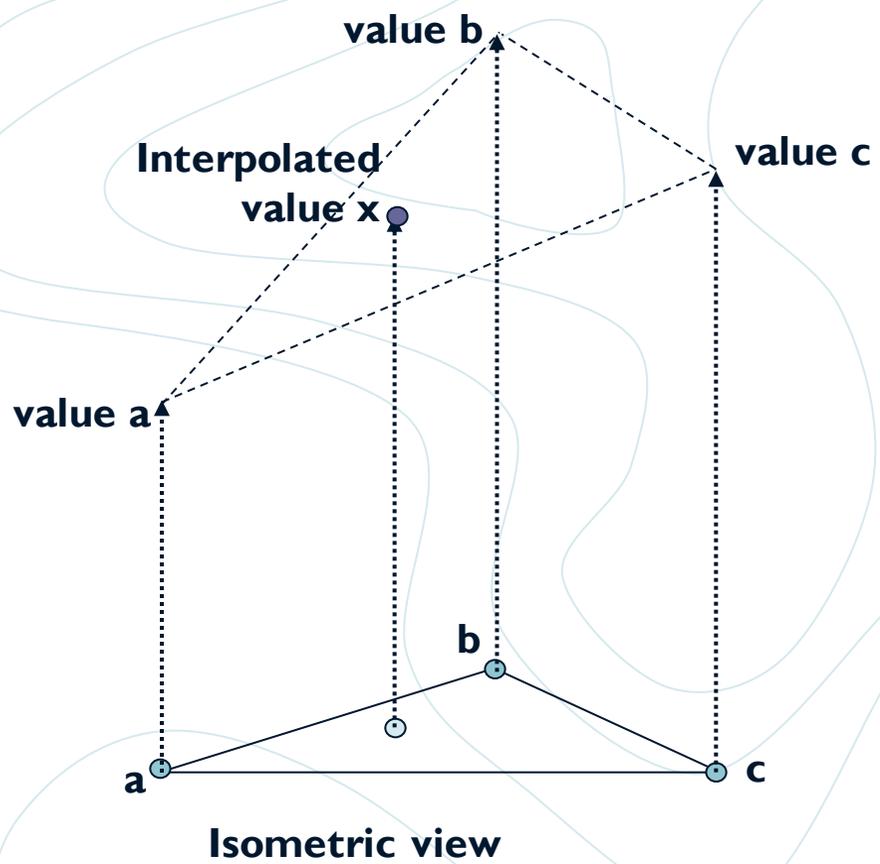
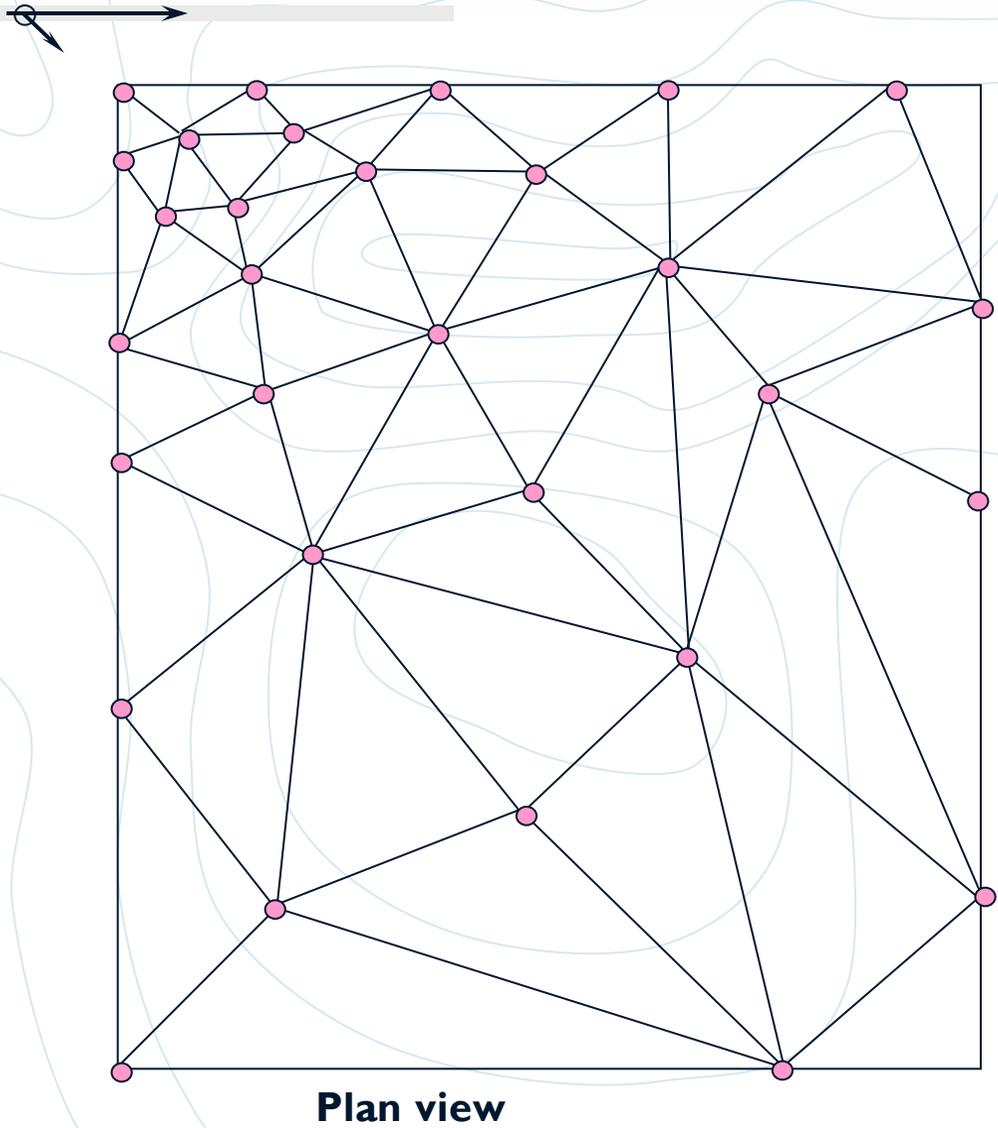
Reverse side of tetradrachm coin representing the pattern of major massifs of the region of Ephesus

(issued between 336-334 BC to pay the persian army)



The same region as depicted by the DEM/SRTM in 2004

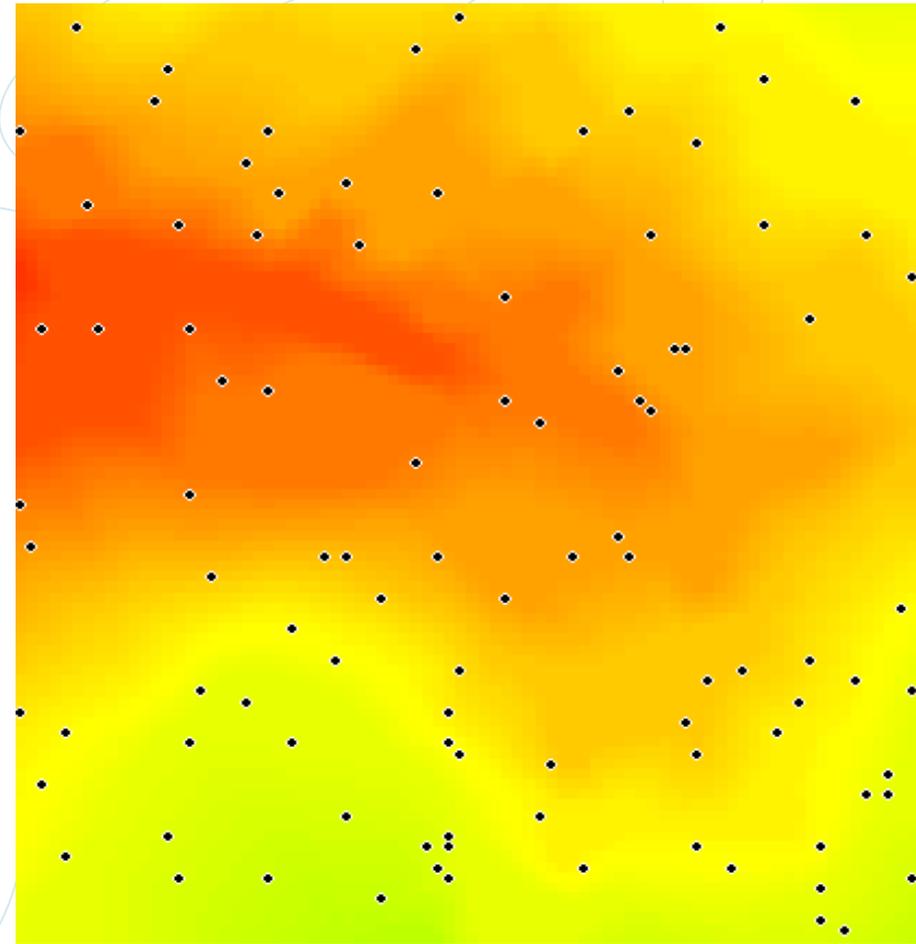
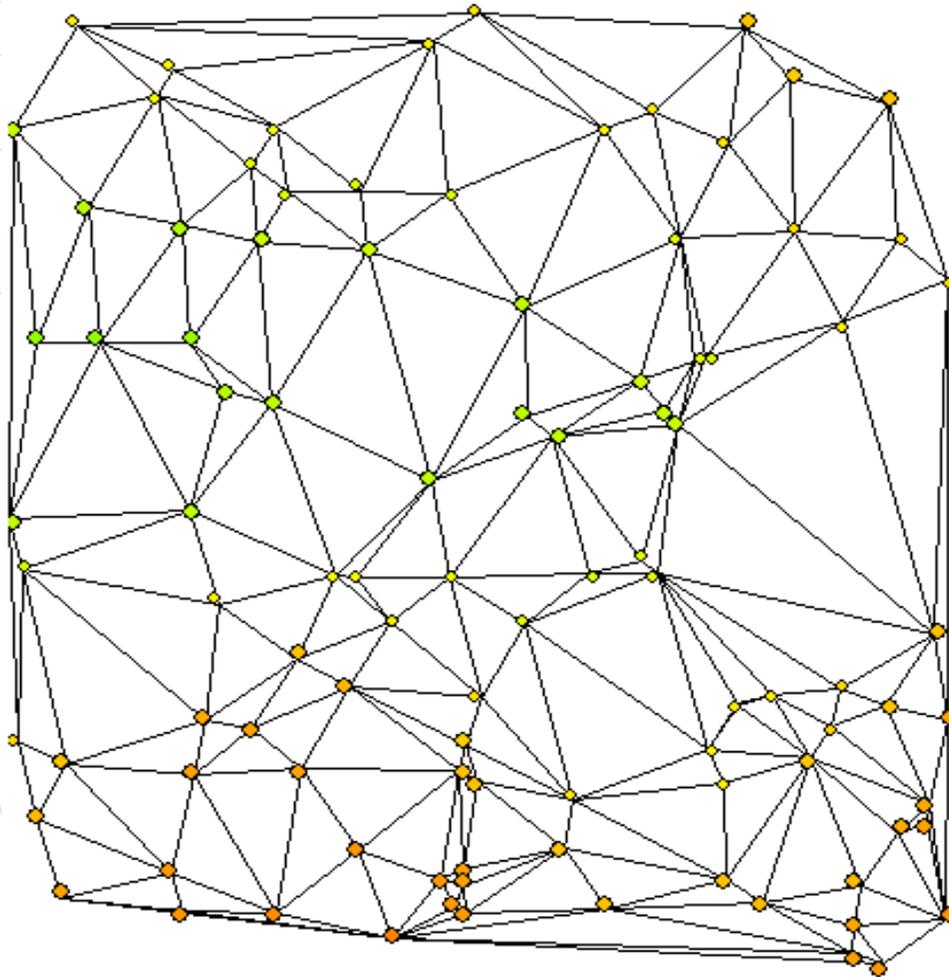
TIN construction



DEMs and TINs

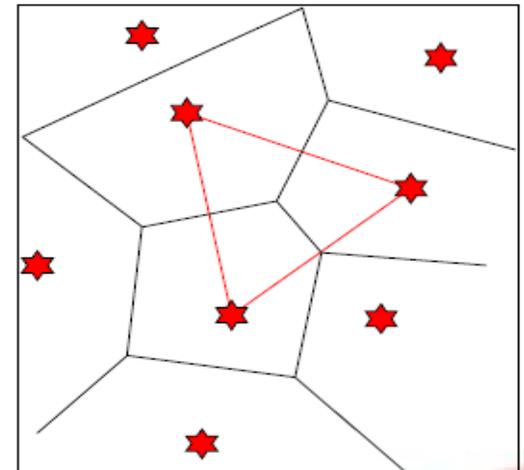
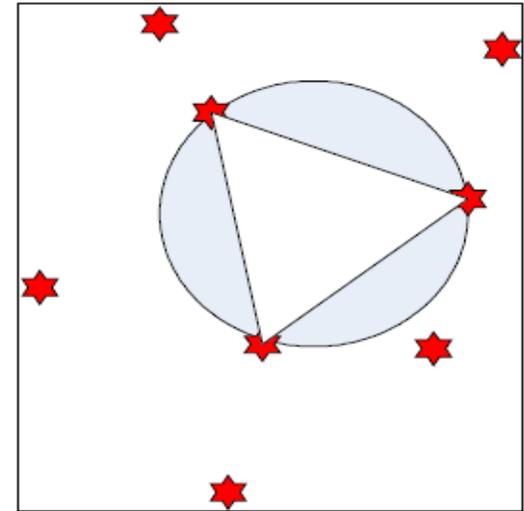
TIN based on same
sample points

DEM with sample points



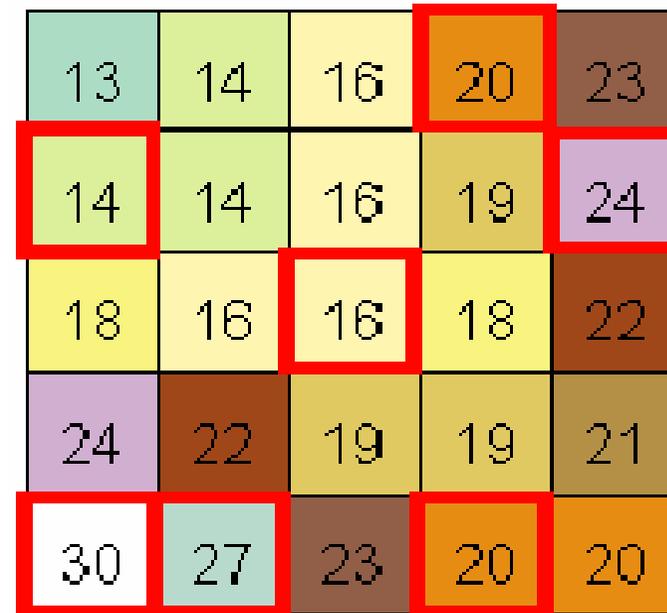
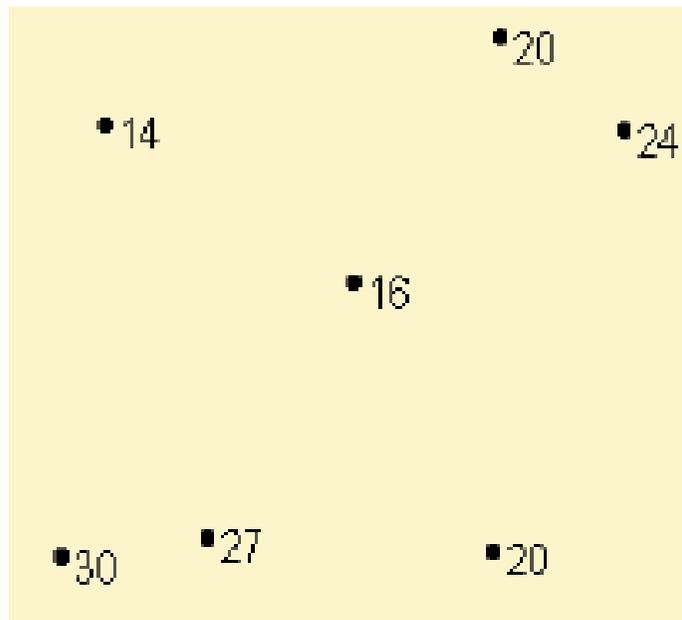
Delaunay Triangulation

- Two approaches
 - Define triangles by finding three points that define a circle that doesn't include any other points
 - Tessellate by assigning all locations to the nearest vertex
 - Boundaries form a set of polygons called Thiessen or Voronoi polygons



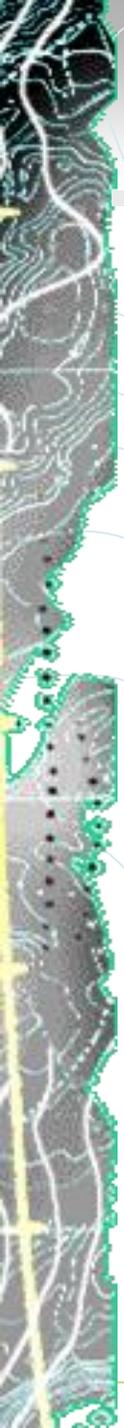
Raster DEM

- Tessellation of regular triangles, **squares** and hexagons
- Simple derivatives, filtering, ...



On the left is a point dataset of known values. On the right is a raster interpolated from these points. Unknown values are predicted with a mathematical formula that uses the values of nearby known points.

Deterministic vs. Stochastic Interpolation

- 
- **Deterministic: Does *not* provide an estimate of errors associated with predicted values (no statistics!)**
 - **Stochastic: Provides and assessment of prediction errors with estimated variances (statistics as a “measure of goodness” of the interpolation)**

Classifying Spatial Interpolation Methods



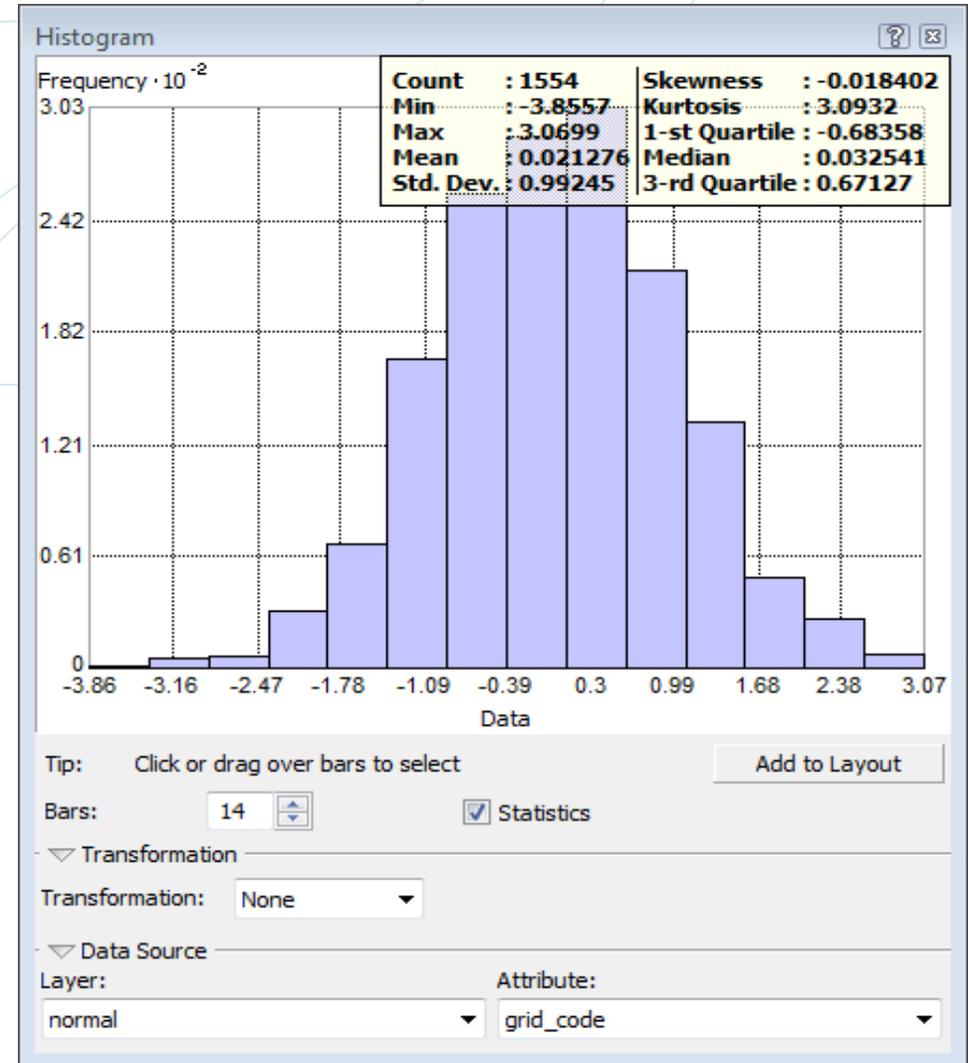
Global		Local	
Deterministic	Stochastic	Deterministic	Stochastic
Trend surface (inexact)*	Regression (inexact)	Thiessen (exact) Density estimation (inexact) Inverse distance weighted (exact) Splines (exact)	Kriging (exact)

*Given some required assumptions, trend surface analysis can be treated as a special case of regression analysis and thus a stochastic method (Griffith and Amrhein 1991).

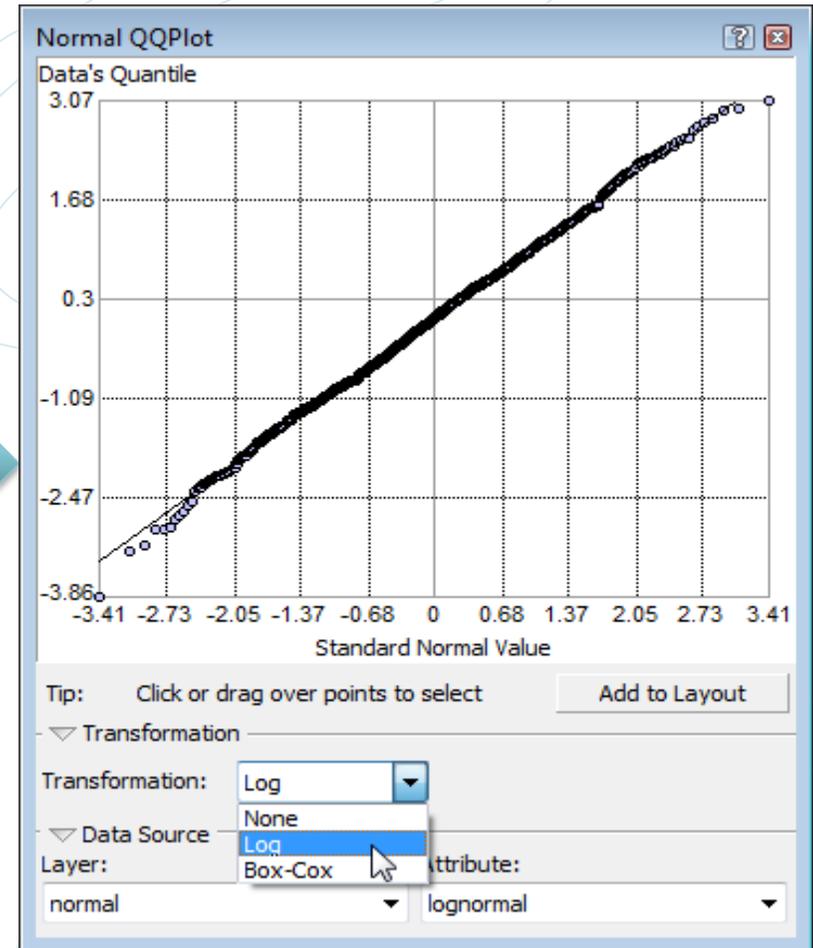
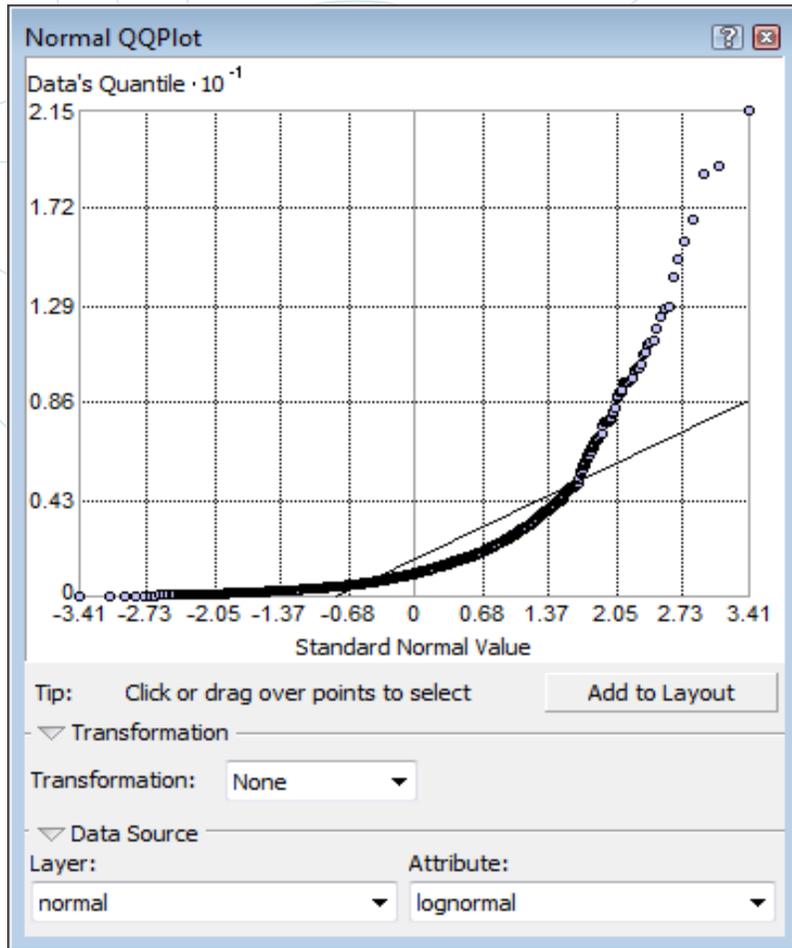
Does my data follow a normal distribution?

- **What should I look for?**

- Bell-shaped
- No outliers
- Mean \approx Median
- Skewness ≈ 0
- Kurtosis ≈ 3



Does my data follow a normal distribution?

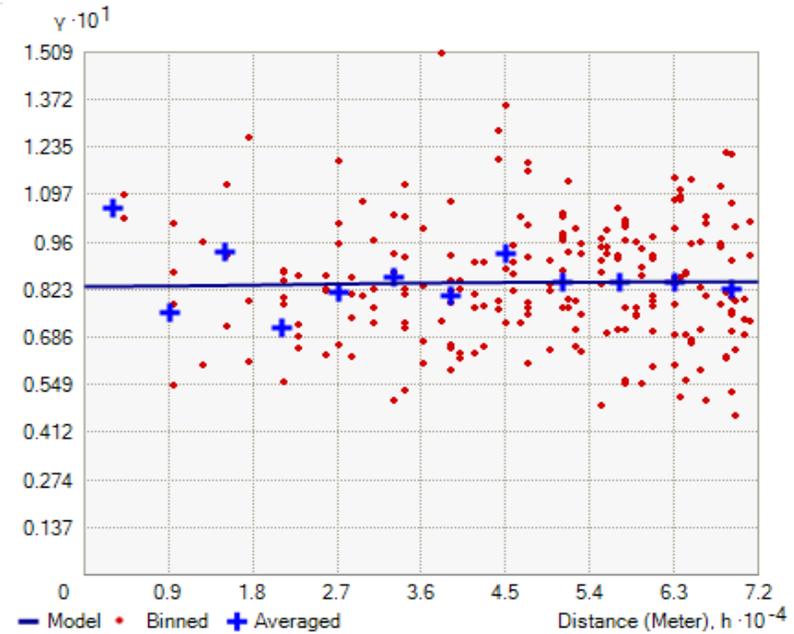
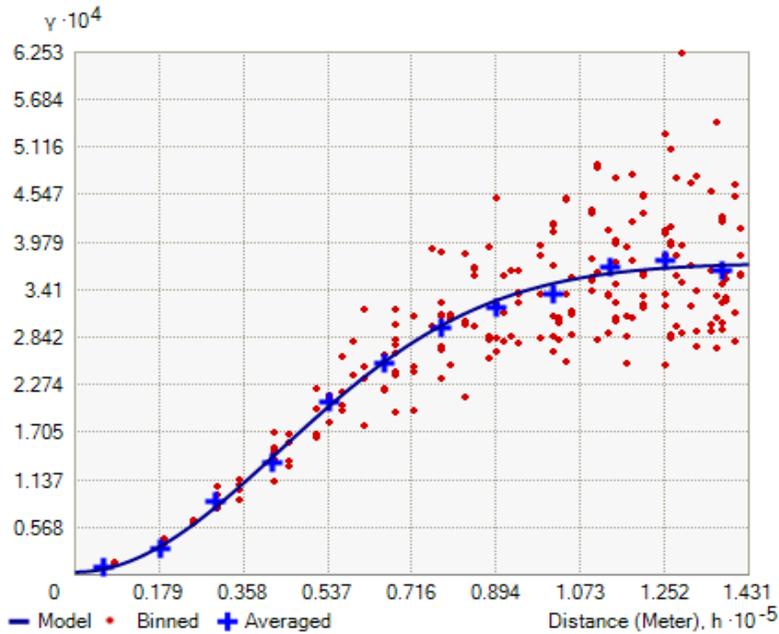


What is autocorrelation?

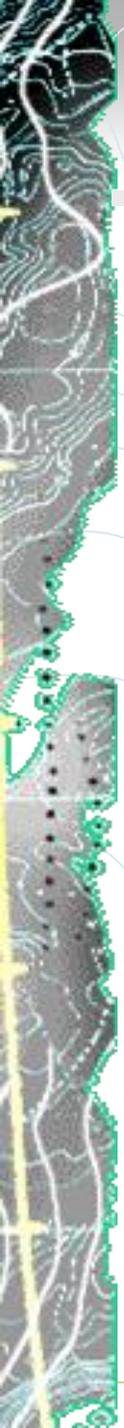


Tobler's first law of geography:

"Everything is related to everything else, but near things are more related than distant things."



Spatial Interpolation in GIS

- 
- **Estimation of z-value for any point location within a specific spatial area**
 - **Assume:**
 - **The data is continuous**
 - **The data are spatially dependent; it can be estimated based on surrounding locations**

Spatial Interpolation

■ Continuous Data

- **Whole Area Methods** – interpolation based on all points in a study area
 - Trend Surface Analysis
 - Fourier Series
- **Local Interpolators** – interpolation may be applied to only a portion of the data
 - Moving Average
 - Splines
 - Kriging

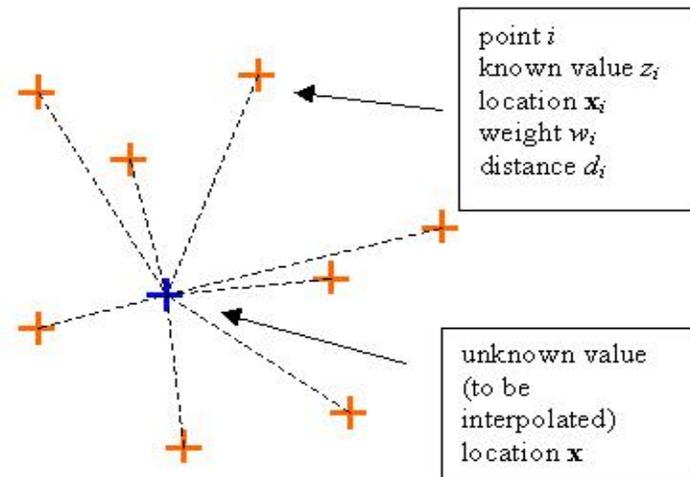
Inverse Distance Weighted

- Each input point has local influence that diminishes with distance
- estimates are averages of values at n known points within window

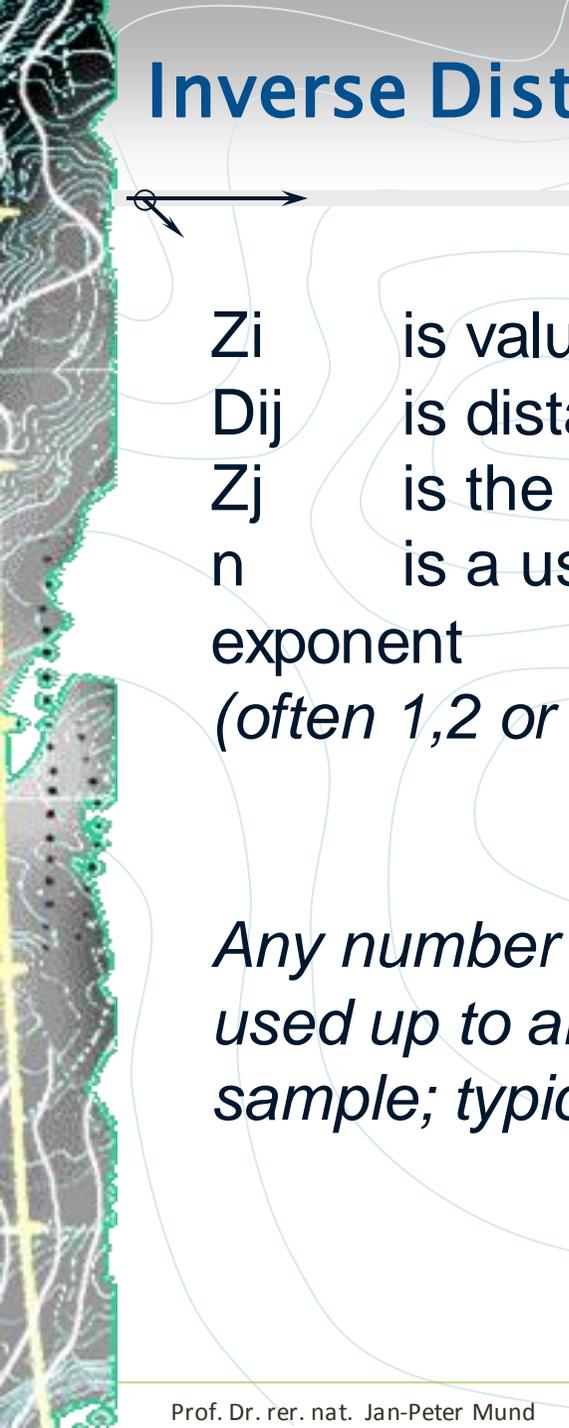
$$z(\mathbf{x}) = \frac{\sum_i w_i z_i}{\sum_i w_i}$$

– where w is some function of distance

$$w_i = 1/d_i^2$$



Inverse Distance Weighting (IDW)



Z_i is value of known point
 D_{ij} is distance to known point
 Z_j is the unknown point
 n is a user selected
exponent
(often 1,2 or 3)

Any number of points may be used up to all points in the sample; typically 3 or more

$$Z_j = \frac{\sum_i \frac{Z_i}{d_{ij}^n}}{\sum_i \frac{1}{d_{ij}^n}}$$

Inverse Distance Weighting

Geostatistical Wizard - IDW Interpolation: Step 1 of 2 - Set Parameters

Optimize Power Value Power: 2

Symbol Size: 8 Method: Neighborhood

Neighbors to Include: 15
 Include at Least: 10

Shape Type:

Shape Angle: 0.0

Major Semiaxis: 4324
 Minor Semiaxis: 4324
 Anisotropy Factor: 1

Test Location
 X: 1947629.1 Y:
 Neighbors : 60
 Estimated = 4.4764

Preview type: Neighbors

< Back Next > Finish

Geostatistical Wizard - IDW Interpolation: Step 2 of 2 - Cross Validation

Chart

Predicted Error

Regression function: $0.446 * x + 1.758$

Prediction Errors

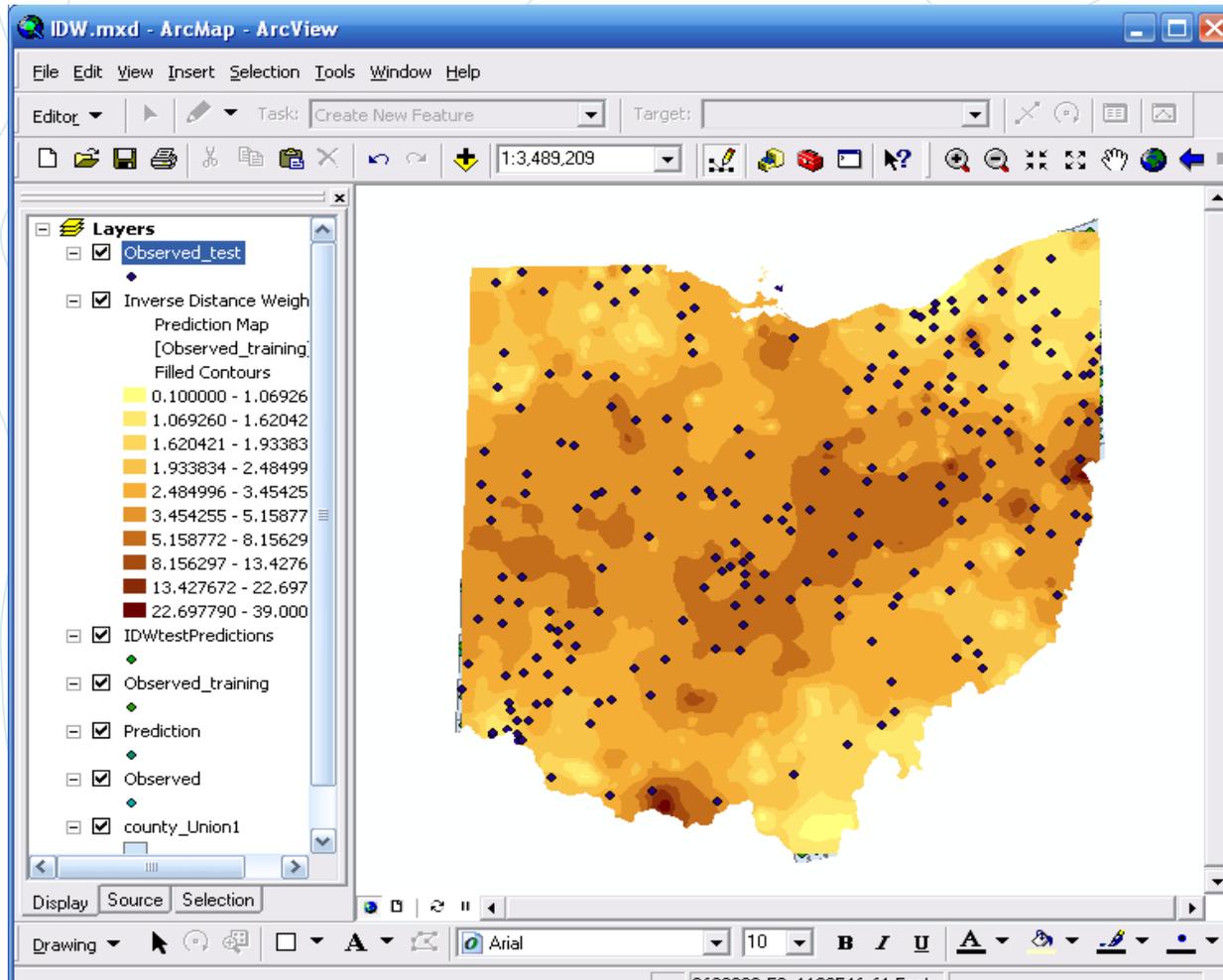
Mean: -0.04331
 Root-Mean-Square: 2.639

Samples: 852 of 852

Included	X	Y	Measured	Predicted
Yes	1354700	586030	5.68	3.3045
Yes	1355200	426510	1.77	2.0274
Yes	1357700	730860	3.74	4.8258
Yes	1361500	468760	3.83	2.0221
Yes	1365900	444910	1.85	2.0752
Yes	1367000	689480	5.25	4.8472
Yes	1368400	814110	5.06	5.3457

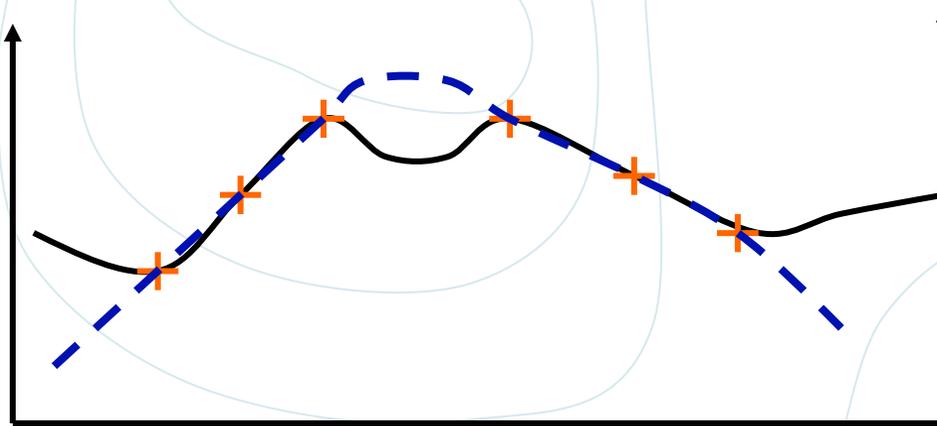
Save Cross Validation... < Back Next > Finish Cancel

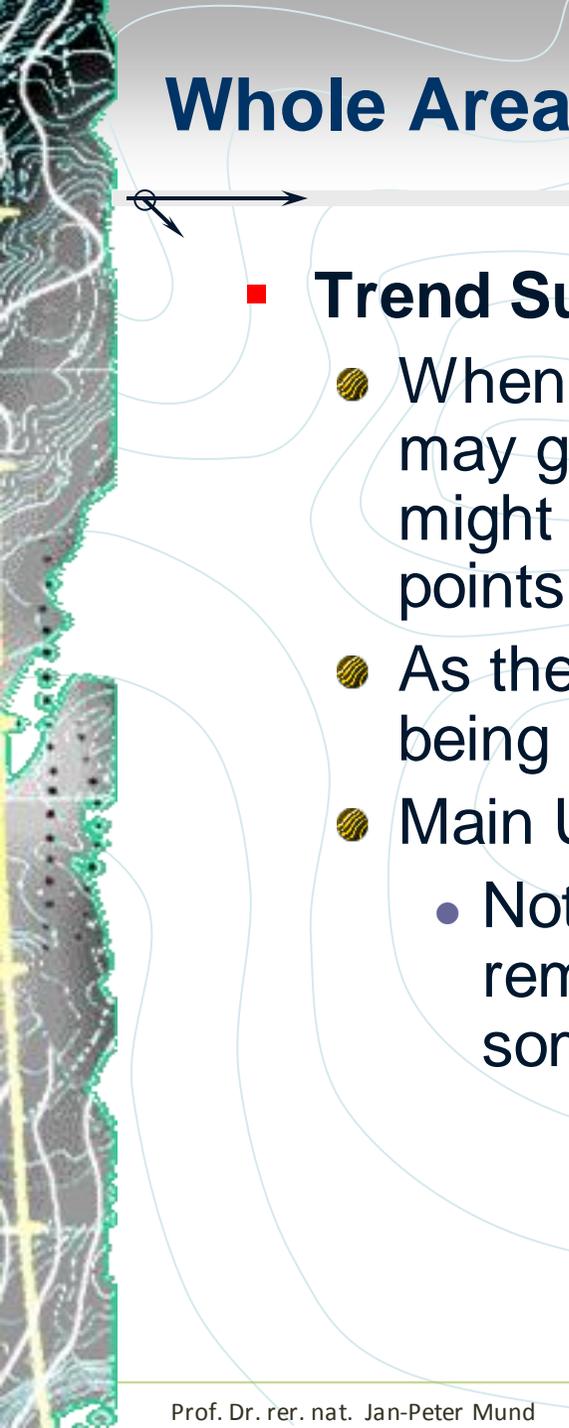
Inverse Distance Weighting



A potentially undesirable characteristic of IDW interpolation

- This set of six data points clearly suggests a hill profile.
- But in areas where there is little or no data the interpolator will move towards the overall mean.
- Blue line shows the profile interpolated by IDW

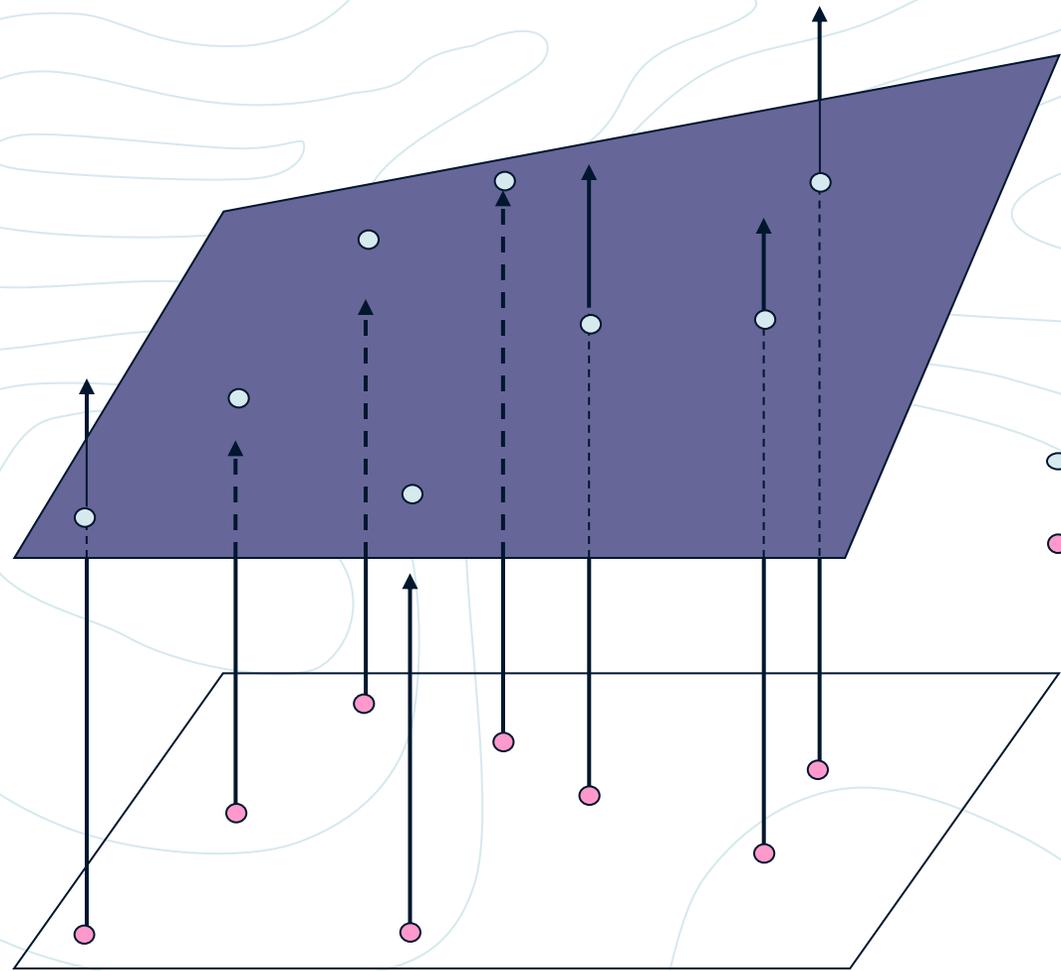




■ Trend Surface

- When an order higher than 1 is used, the interpolator may generate a Grid whose minimum and maximum might exceed the minimum and maximum of the input points
- As the order of the polynomial is increased, the surface being fitted becomes progressively more complex
- Main Use
 - Not an interpolator within a region, but a way of removing broad features of the data prior to using some other local interpolator

Fitting a single polynomial trend surface

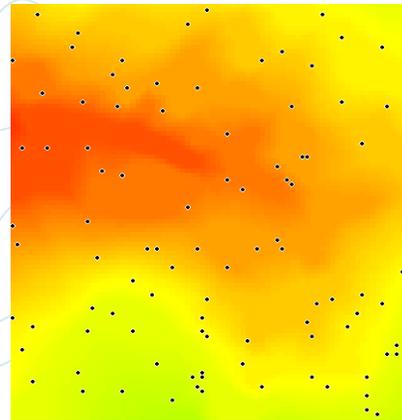


- interpolated point
- data point

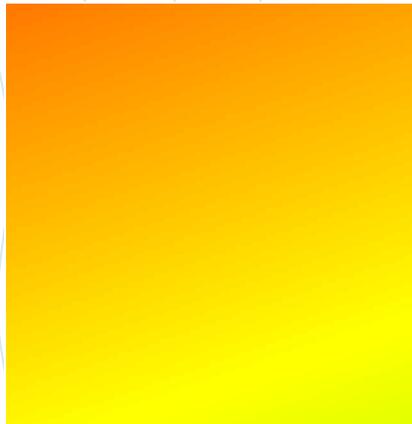
Example trend surfaces



Source surface with
sample points

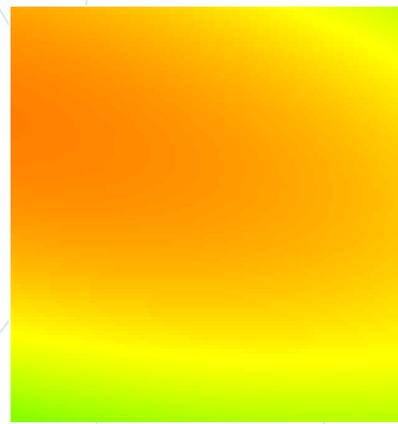


Linear



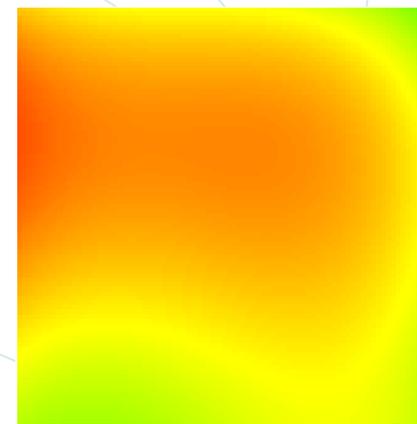
Goodness of fit
(R2) = 45.42 %

Quadratic



Goodness of fit
(R2) = 82.11 %

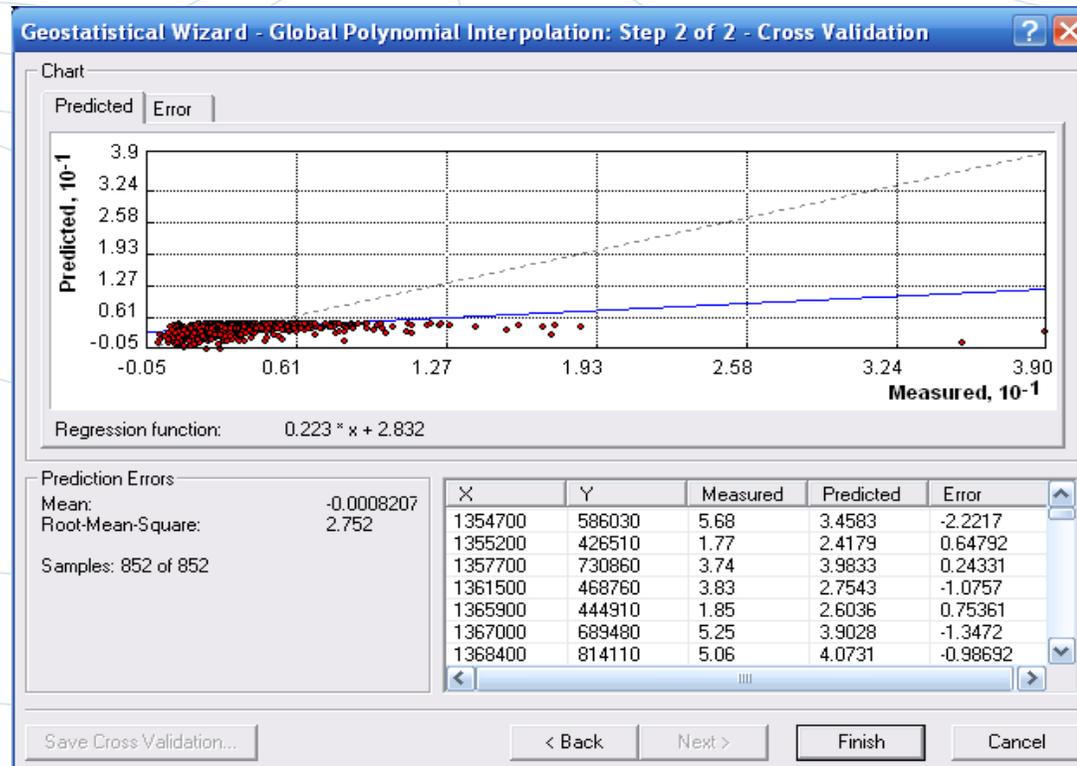
Cubic



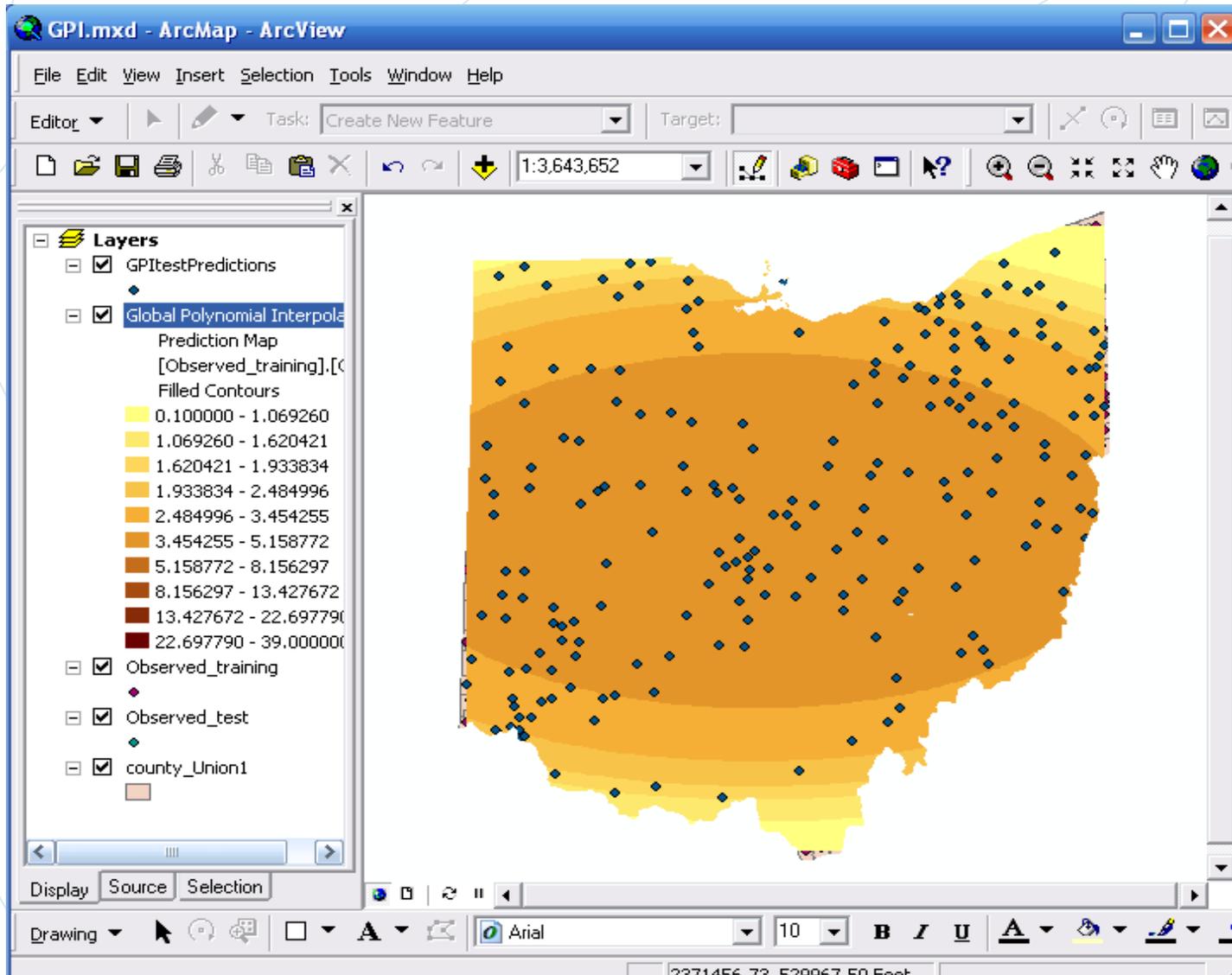
Goodness of fit
(R2) = 92.72 %

Global Polynomial Interpolation

- Global polynomial interpolation technique fits a plane through the measured data points. A plane is typically a polynomial.

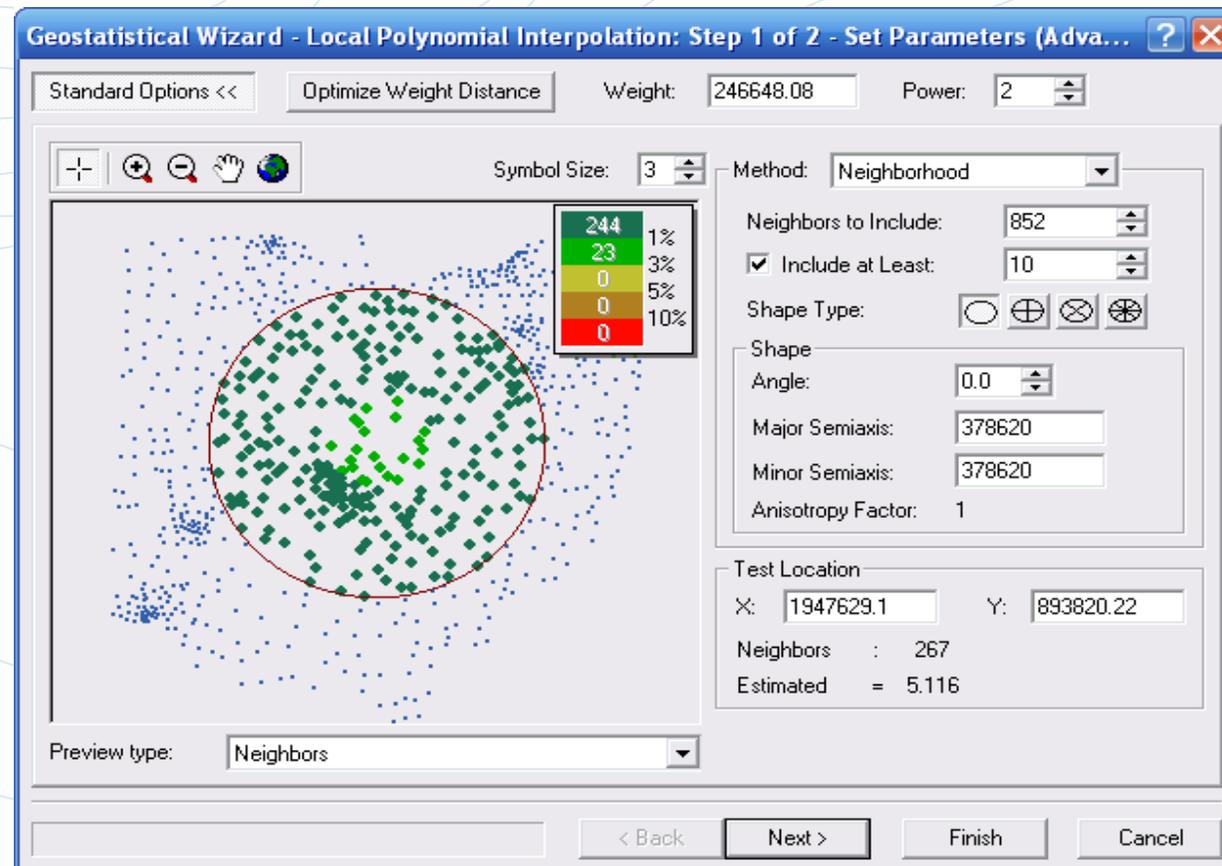


Global Polynomial Interpolation

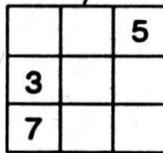
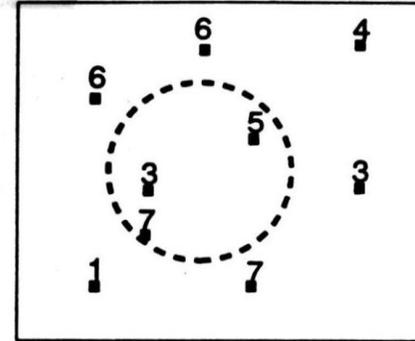
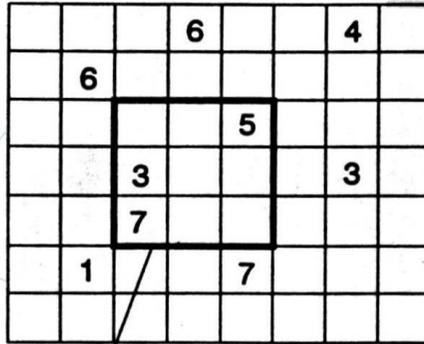


Local polynomial Interpolation

- While Global Polynomial interpolation fits a polynomial to the entire surface, Local Polynomial interpolation fits many polynomials, each within specified overlapping neighborhoods.

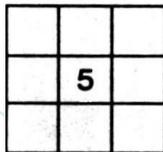


Spatial moving average (SMA)

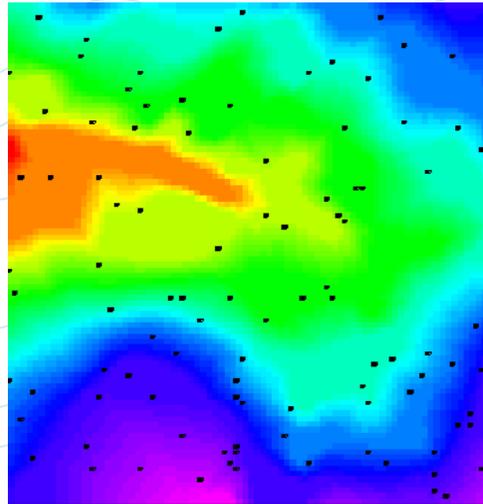


$$\frac{3 + 7 + 5}{3} = 5$$

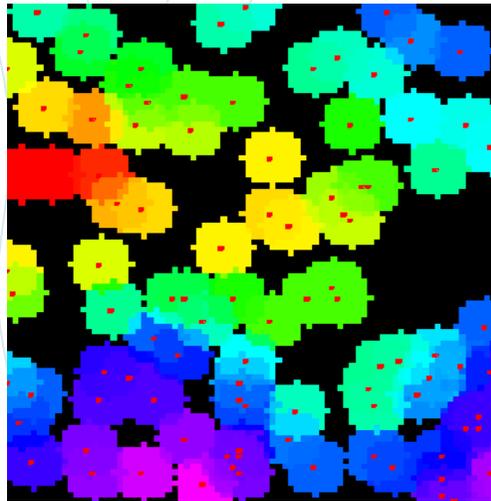
$$\frac{3 + 7 + 5}{3} = 5$$



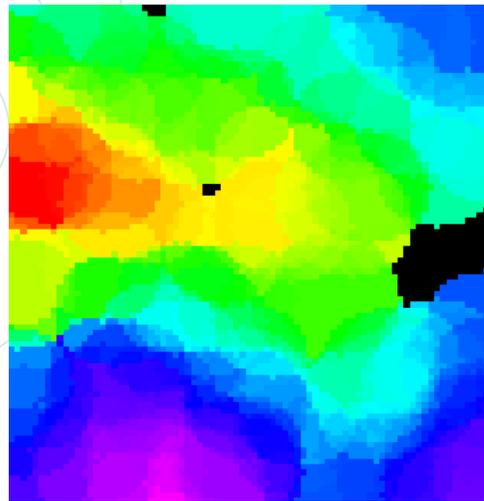
Example SMA (circular filter)



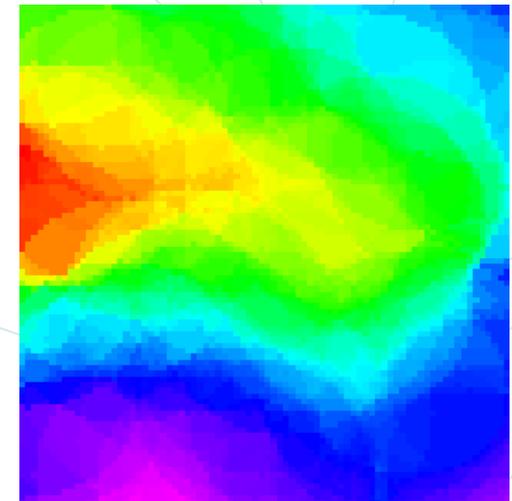
Source surface with sample points



11x11 circular filter SMA with sample points



21x21 circular filter SMA



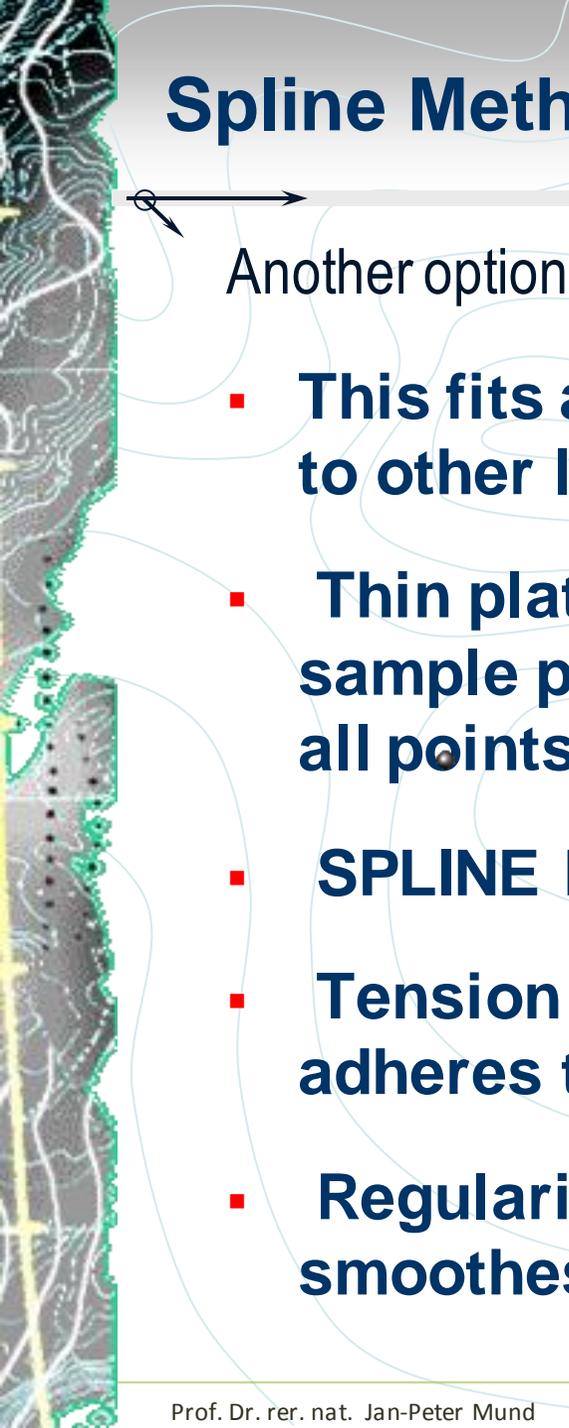
41x41 circular filter SMA

Evaluation Criteria

- **Several statistical indicators (Root Mean Square Error (RMSE), Mean Error (ME), Mean Absolute Error (MAE) and Mean Square Error (MSE)) are computed on observed and predicted radon concentrations.**

- **Confidence limits on the statistics for**
 - **Normalized Mean Square Error (NMSE),**
 - **Fractional Bias (FB), and**
 - **Coefficient of Correlation (r)**
- **are calculated using Bootstrap application to identify the most suitable interpolation technique.**

Spline Method



Another option for interpolation method:

- **This fits a curve through the sample data assign values to other locations based on their location on the curve**
- **Thin plate splines create a surface that passes through sample points with the least possible change in slope at all points, that is with a minimum curvature surface**
- **SPLINE has two types: regularized and tension**
- **Tension results in a rougher surface that more closely adheres to abrupt changes in sample points**
- **Regularized results in a smoother surface that smoothes out abruptly changing values somewhat**

Splines

- 
- Name derived from the drafting tool, a flexible ruler, that helps create smooth curves through several points
 - Spline functions (also called splines) are used to interpolate along a smooth curve. (similar to the flexible ruler).
 - Fits a minimum-curvature surface through the input points. Conceptually, it is like bending a sheet of rubber to pass through the points, while minimizing the total curvature of the surface
 - Force a smooth line to pass through a desired set of points
 - Constructed from a set of joined polynomial functions

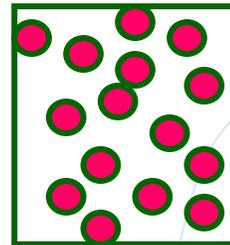
Kriging

- 
- **Similar to Inverse Distance Weighting (IDW)**
 - **Kriging uses the minimum variance method to calculate the weights rather than applying an arbitrary or less precise weighting scheme**
 - **A set of sample points are used to estimate the shape of the Variogram model is made**
 - **A line is fit through the set of semi-variance points**
 - **The Variogram model is then used to interpolate the entire surface**

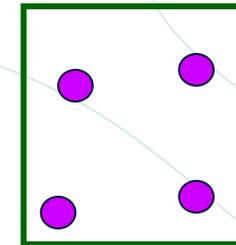
Kriging assumptions

- Some spatial surfaces cannot be modeled using deterministic methods that use smooth mathematical functions.
- Specifically if data are sparse, for example ground-water modeling, gravity data, soil mapping, water toxicity, air pollution, bathymetric data etc.
- Kriging is a stochastic interpolation method in contrast with deterministic methods (TIN, Inverse distance, trend estimation)
- It attempts to statistically obtain the optimal prediction i.e. to provide the Best Linear Unbiased Estimation (BLUE), specifically when data are sparse

Dense => deterministic



Sparse => kriging



Kriging

- Lag distance
- Where:
- Z_i is a variable at a sample point
- h_i is the distance between sample points

- Every possible set of pairs Z_i, Z_j defines a distance h_{ij} , and is different by the amount
- $Z_i - Z_j$.

- The distance h_{ij} is known as the lag distance between point i and j . Also there is a subset of points in a sample set that are a given lag distance apart

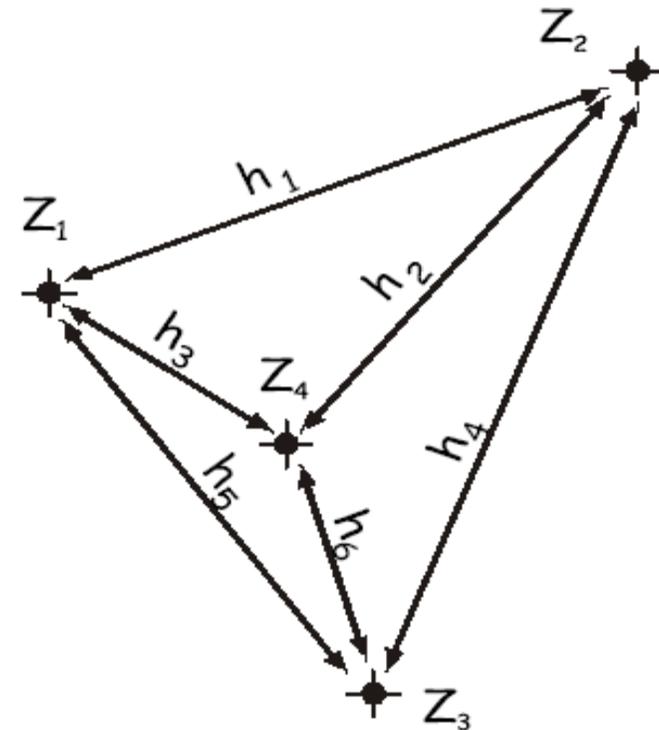


Figure 9-12: Lag distances, used in calculating semi-variances for kriging.

Kriging



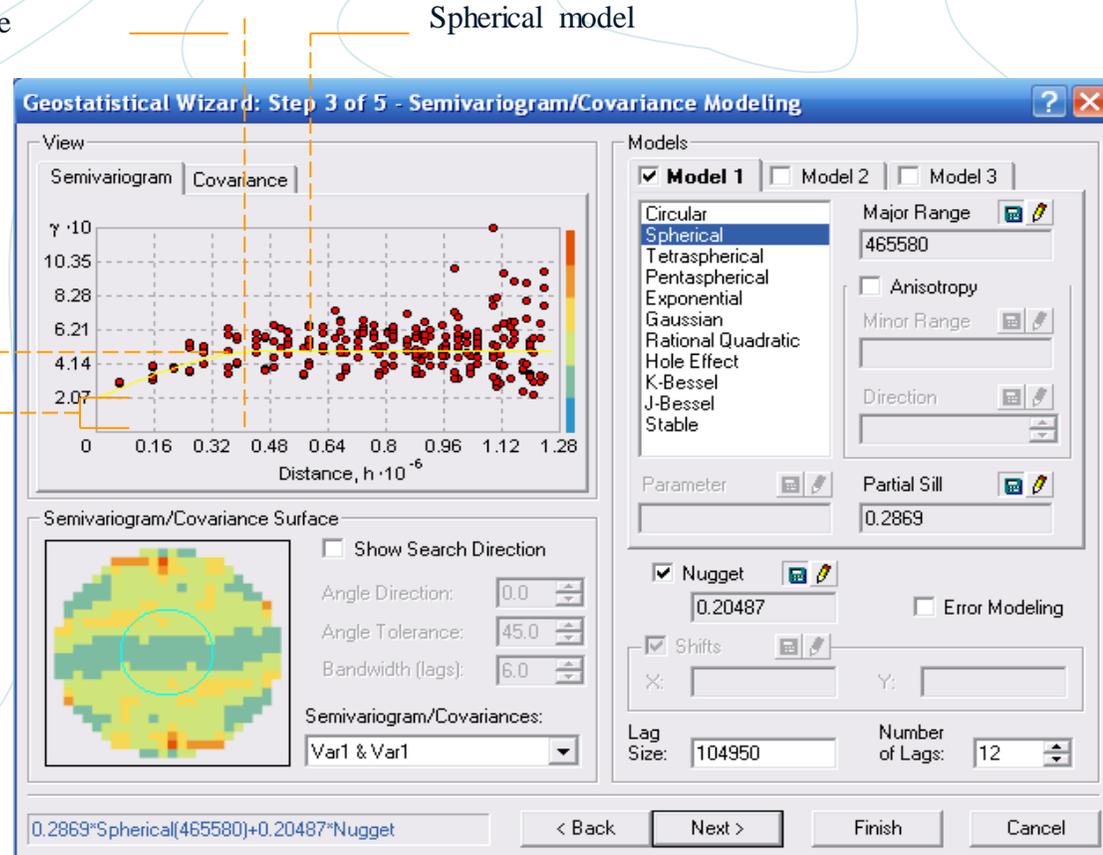
- **Semi-variance**

$$\gamma(h) = 1/2n * \sum (Z_i - Z_j)^2$$

- **Where Z_i is the measured variable at one point**
- **Z_j is another at h distance away**
- **n is the number of pairs that are approximately h distance apart**
- **Semi-variance may be calculated for any h**
- **(When nearby points are similar ($Z_i - Z_j$) is small so the semi-variance is small. High spatial autocorrelation means points near each other have similar Z values)**

Ordinary Kriging

- There are three primary parameters that describe the autocorrelation of radon concentrations. These are range, nugget and sill.
 - The range is where the best-fit line starts to level off, (46.55). Within the range, all data are correlated.
 - The maximum semivariogram value is sill parameter (0.2869)
 - Nugget is data variation due to measurement errors (0.20487).



Ordinary Kriging

Geostatistical Wizard: Step 4 of 5 - Searching Neighborhood

Dataset Selection: Dataset 1

Symbol Size: 3

Method: Neighborhood

Neighbors to Include: 5

Include at Least: 2

Shape Type:

Shape

Angle: 0.0

Major Semiaxis: 465580

Minor Semiaxis: 465580

Anisotropy Factor: 1

Test Location
X: 1947629.1 Y: 795650.48

Neighbors: 20
Prediction = 4.8685

Preview type: Neighbors

< Back Next > Finish Cancel

Geostatistical Wizard: Step 5 of 5 - Cross Validation

Chart

Predicted | Error | Standardized Error | QQPlot

Regression function: $0.483 * x + 1.681$

Prediction Errors

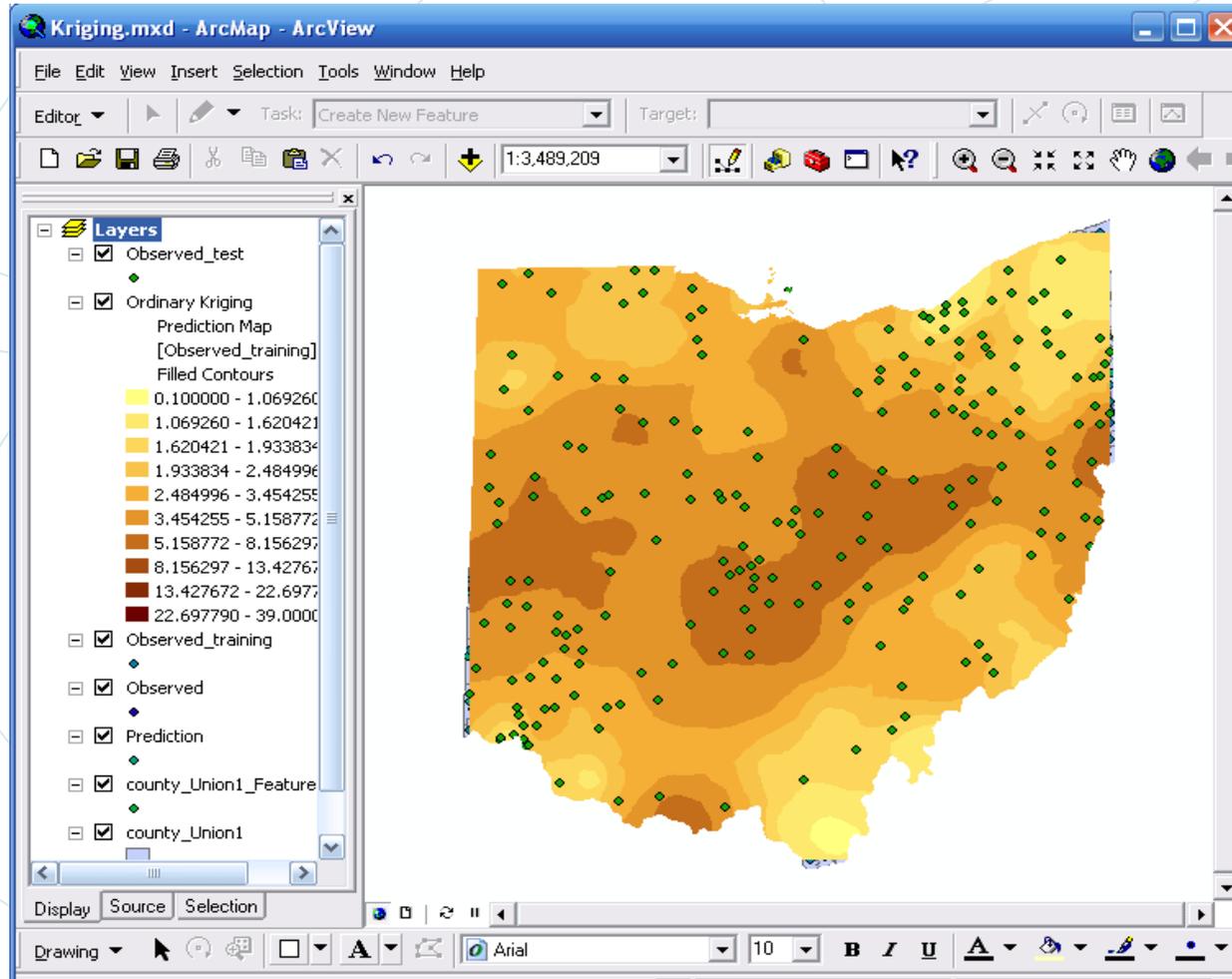
Mean: -0.05196
Root-Mean-Square: 2.573
Average Standard Error: 2.44
Mean Standardized: -0.03232
Root-Mean-Square Standardized: 1.111

Samples: 852 of 852

Included	X	Y	Measured	Predicted
Yes	1354700	586030	5.68	4.1177
Yes	1355200	426510	1.77	2.4489
Yes	1357700	730860	3.74	5.5739
Yes	1361500	468760	3.83	2.4825
Yes	1365900	444910	1.85	2.5598
Yes	1367000	689480	5.25	5.1353
Yes	1368400	814110	5.06	5.1127

Save Cross Validation... < Back Next > Finish Cancel

Ordinary Kriging



Analysis of Variogram

Geostatistical Wizard: Step 2 of 4 - Semivariogram/Covariance Modeling

View

Semivariogram | Covariance

$\gamma \cdot 10^4$

Distance, $h \cdot 10^{-5}$

Semivariogram/Covariance Surface

Show Search Direction

Angle Direction: 0.0

Angle Tolerance: 45.0

Bandwidth (lags): 6.0

Semivariogram/Covariances: Var1 & Var1

Models

Model 1 | Model 2 | Model 3

Circular
Spherical
Tetraspherical
Pentaspherical
Exponential
Gaussian
Rational Quadratic
Hole Effect
K-Bessel
J-Bessel
Stable

Major Range: 168410

Anisotropy

Minor Range: []

Direction: []

Parameter: []

Partial Sill: 0.00026232

Nugget: 0.00018958 | Error Modeling

Shifts

X: [] Y: []

Lag Size: 33704 | Number of Lags: 12

0.00026232*Spherical(168410)+0.00018958*Nugget

< Back | Next > | Finish | Cancel

How Many Neighbors?

Geostatistical Wizard: Step 3 of 4 - Searching Neighborhood

Dataset Selection: Dataset 1

Symbol Size: 3

Method: Neighborhood

Neighbors to Include: 40

Include at Least: 2

Shape Type: Shape

Angle: 0.0

Major Semiaxis: 168410

Minor Semiaxis: 168410

Anisotropy Factor: 1

Test Location

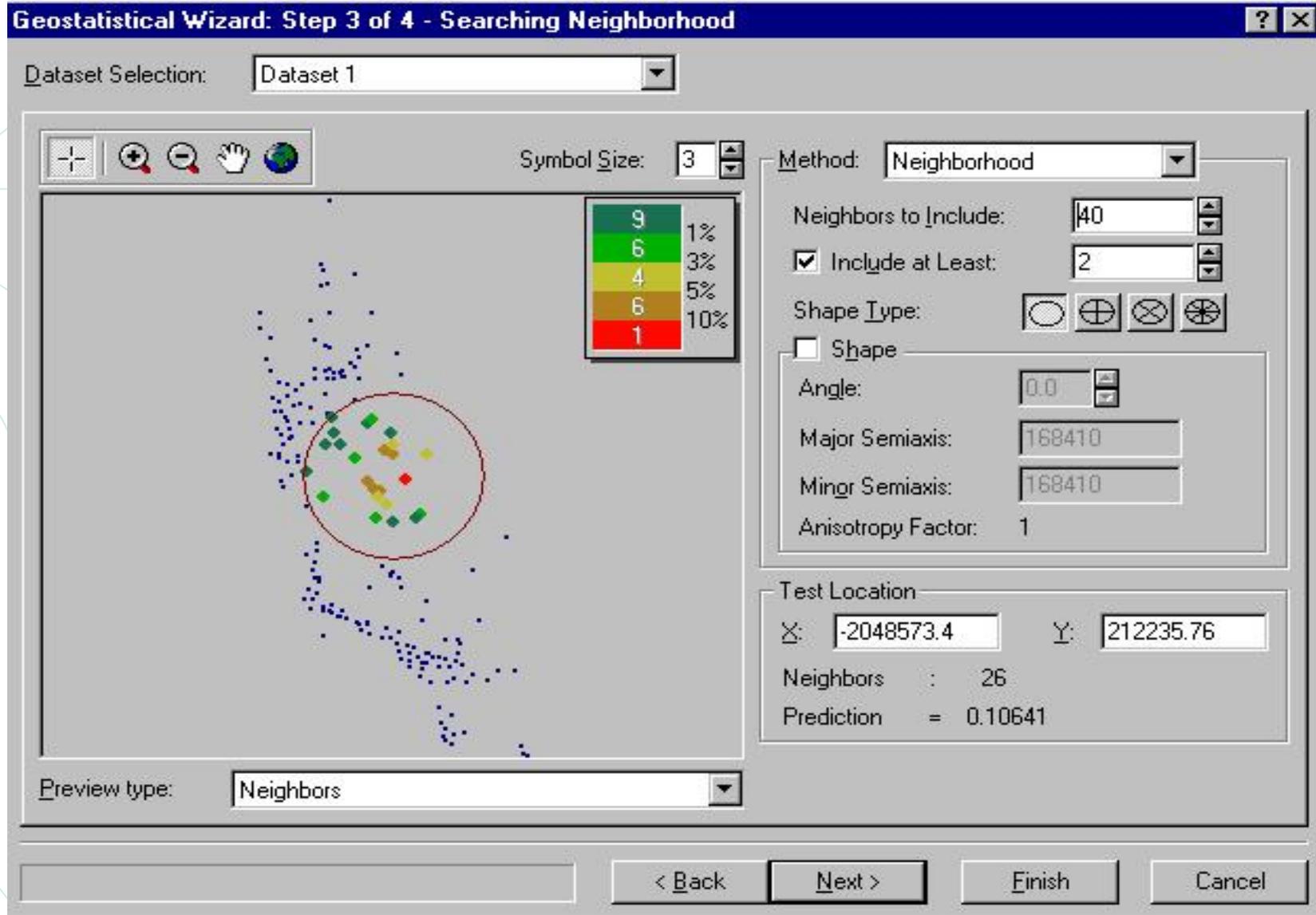
X: -2048573.4 Y: 212235.76

Neighbors : 26

Prediction = 0.10641

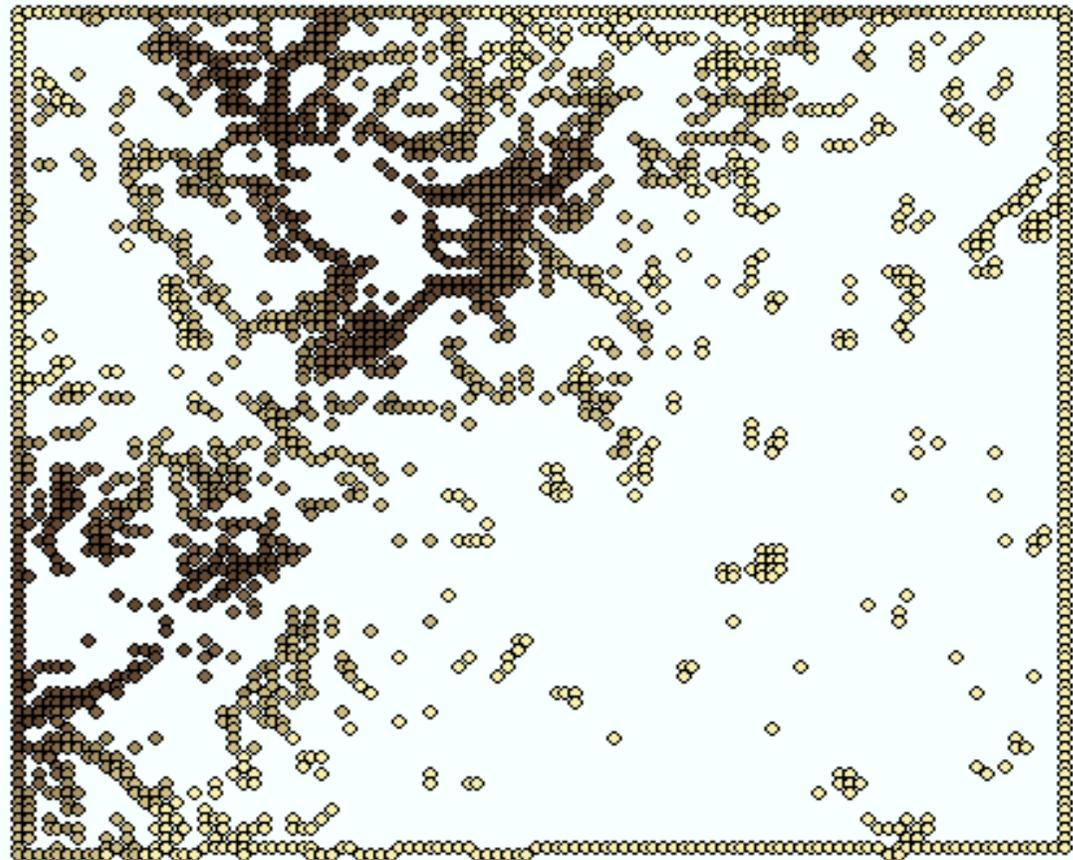
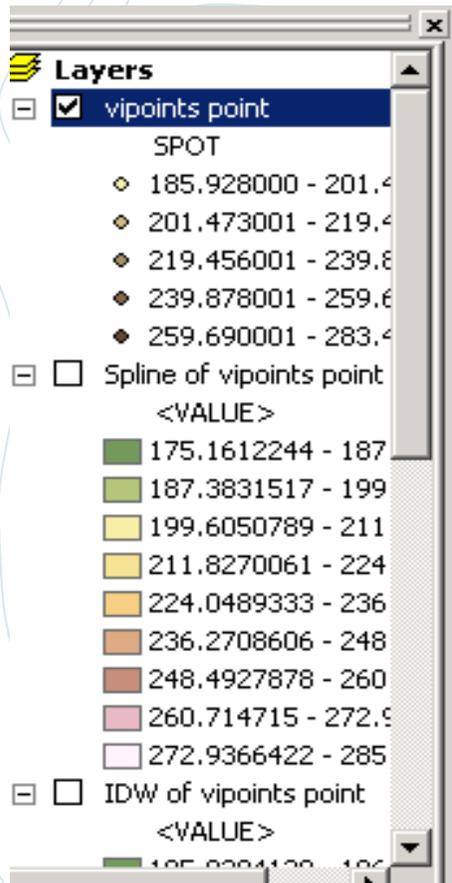
Preview type: Neighbors

< Back Next > Finish Cancel



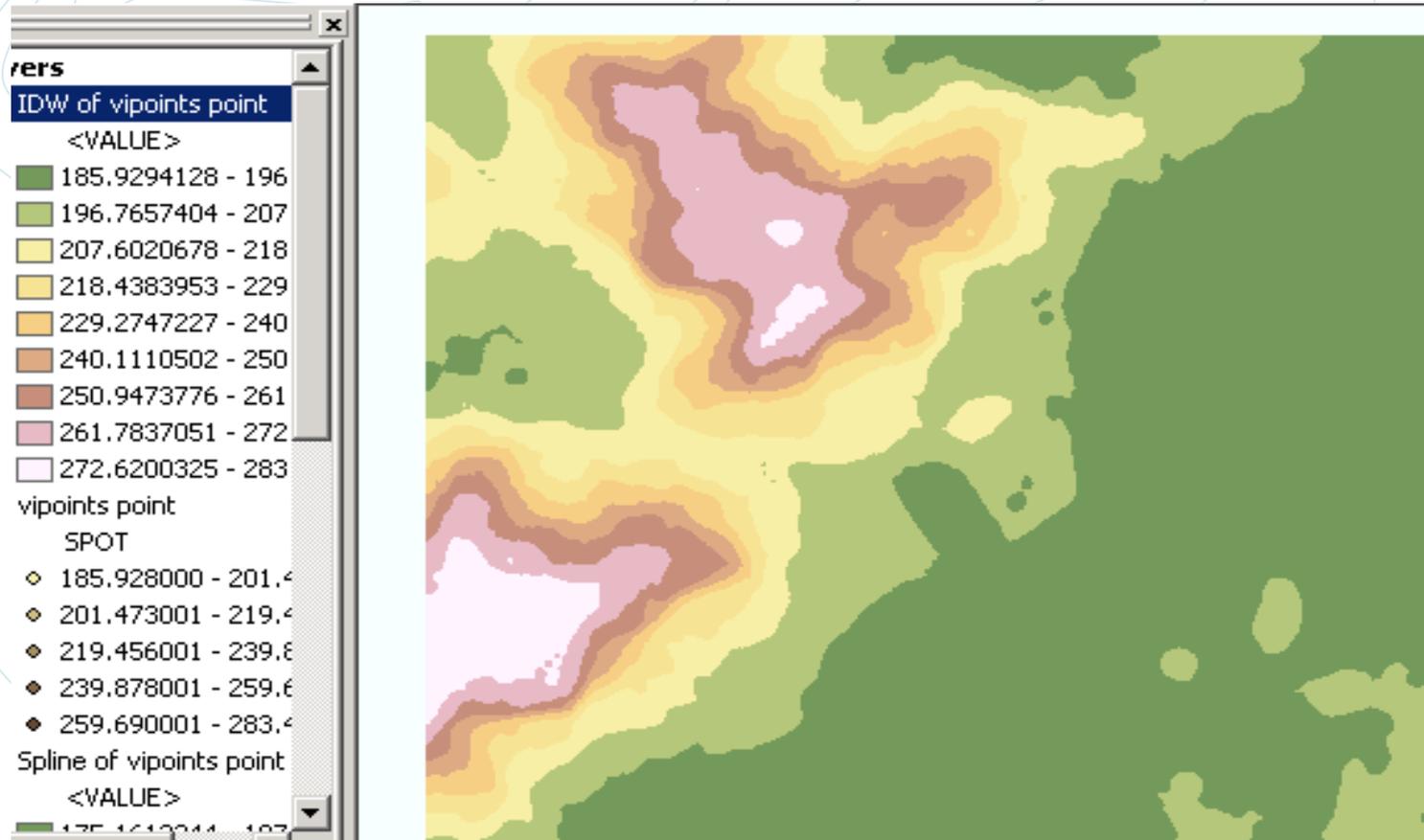
Example

- Here are some sample elevation points from which surfaces were derived using the three methods



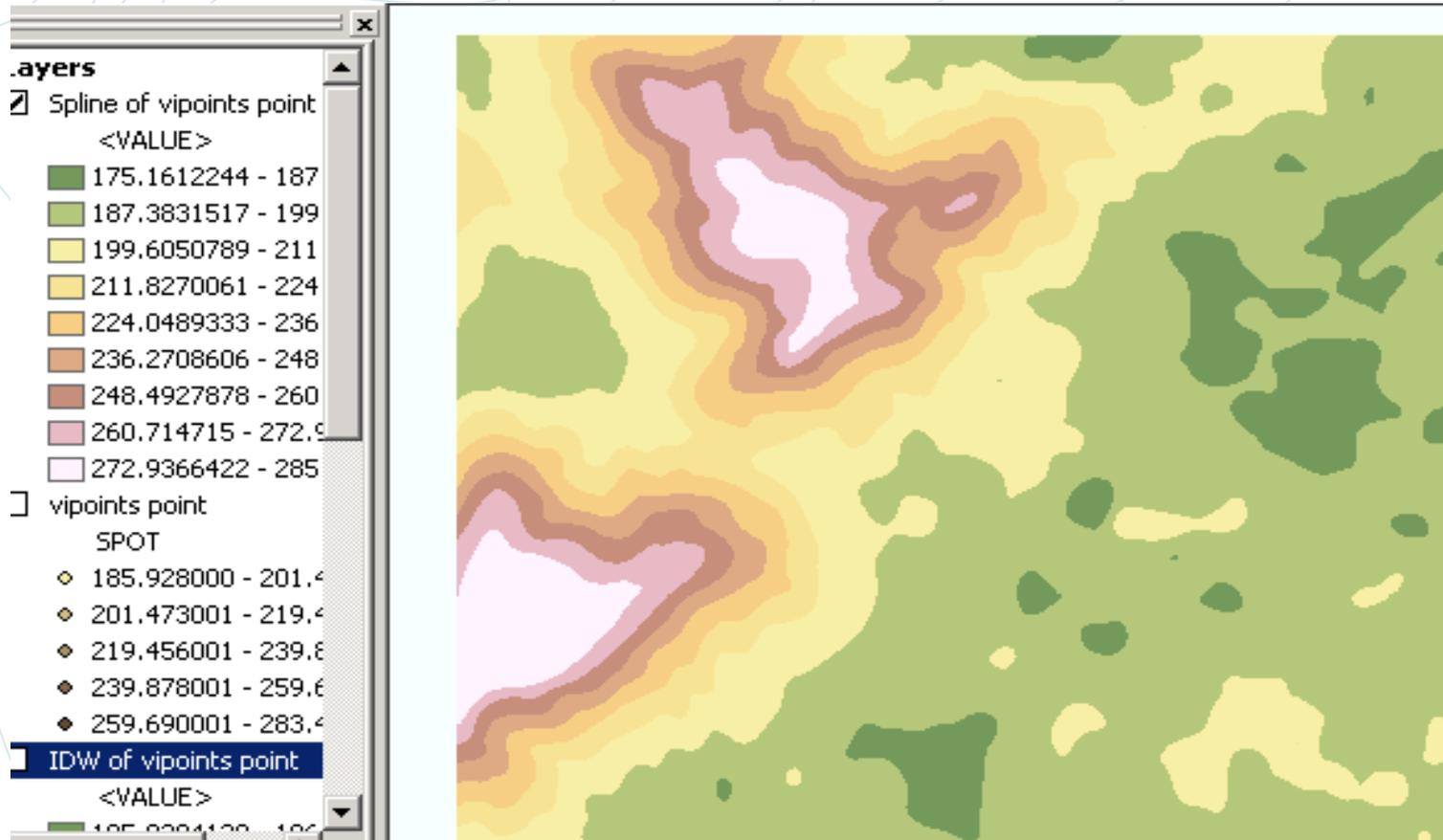
Example: IDW

- Done with $P = 2$. Notice how it is not as smooth as Spline. This is because of the weighting function introduced through P



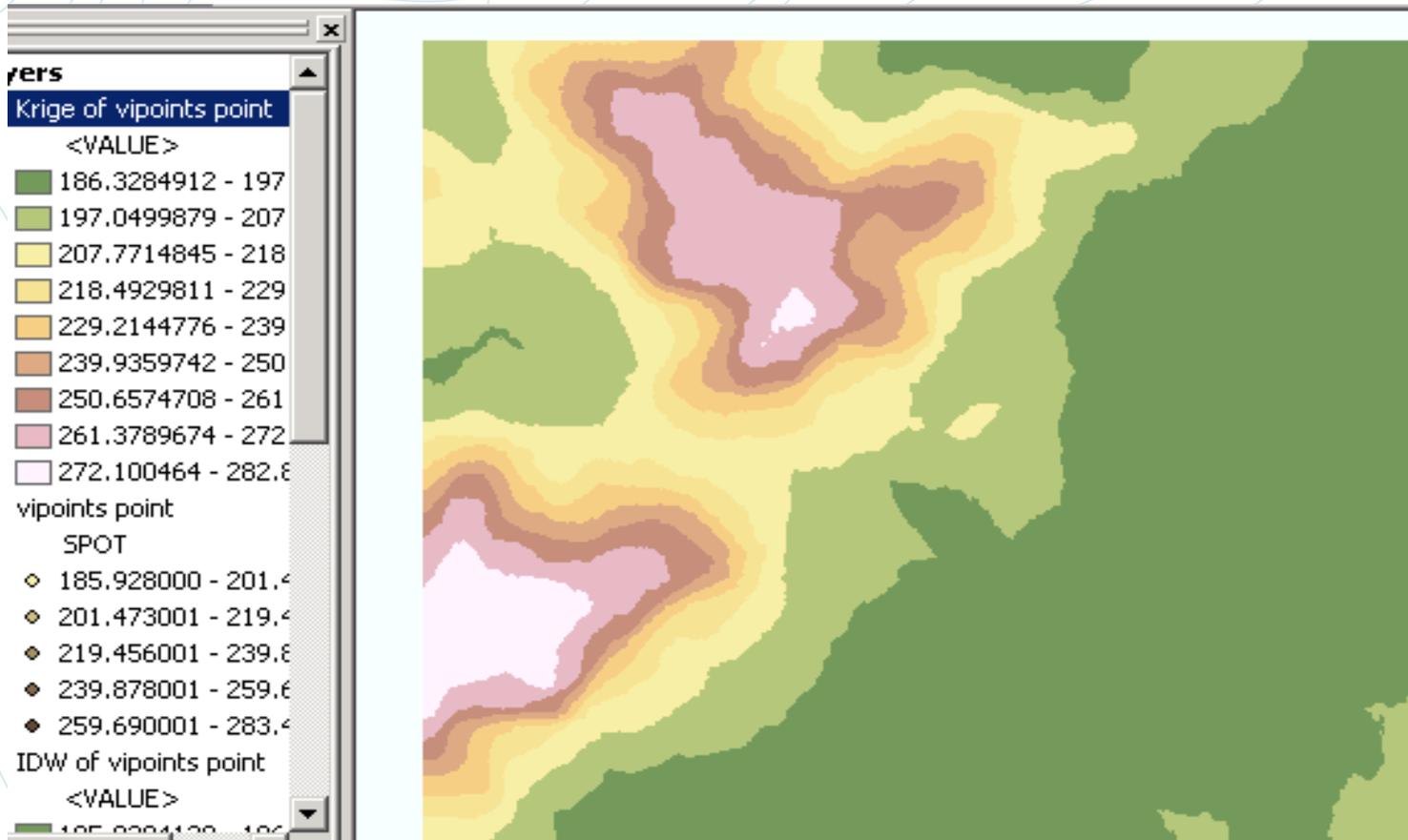
Example: Spline

- Note how smooth the curves of the terrain are; this is because Spline is fitting a simply polynomial equation through the points



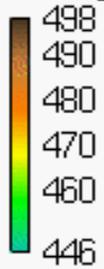
Example: Kriging

- This one is kind of in between—because it fits an equation through point, but weights it based on probabilities

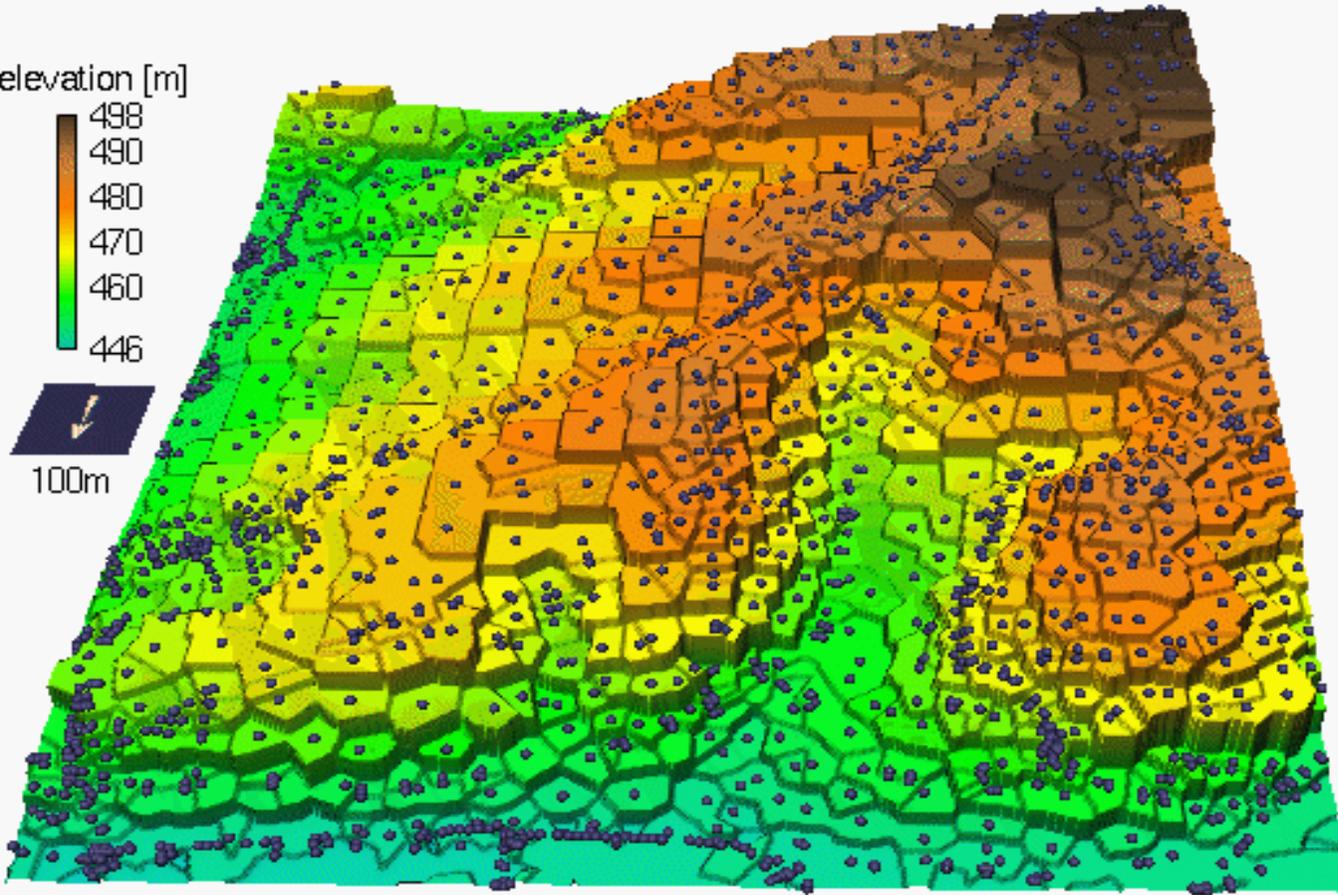


Theissen

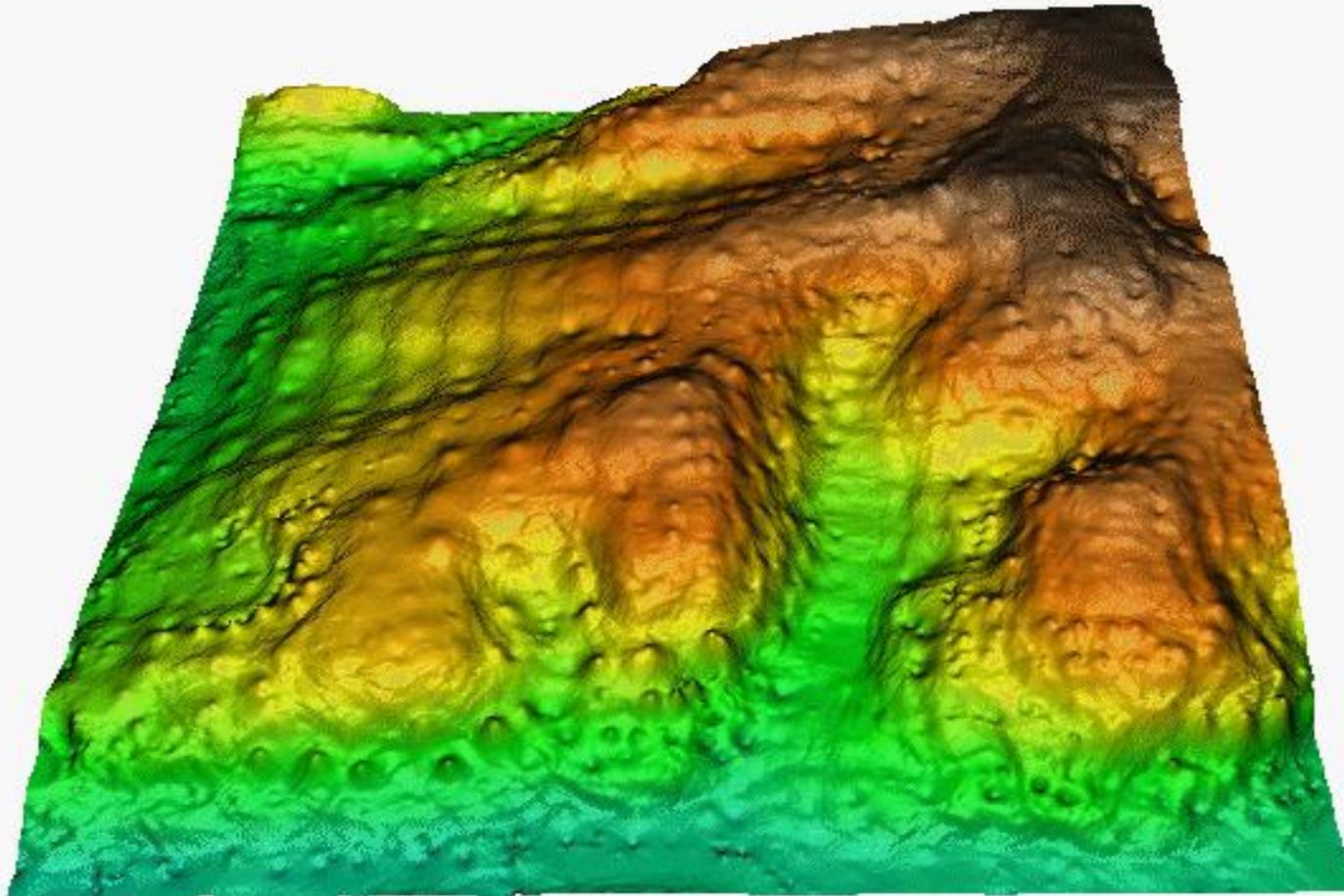
elevation [m]



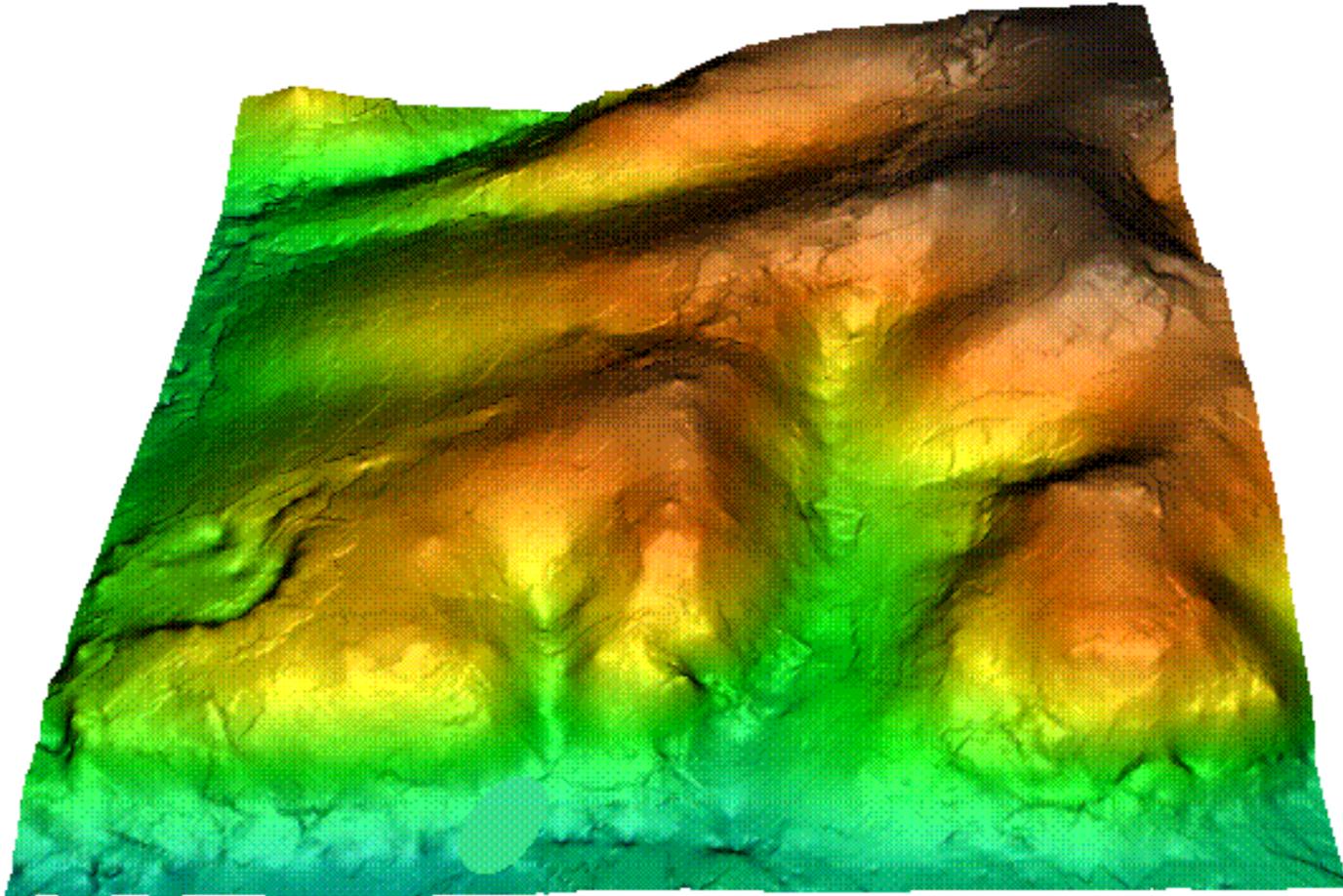
100m



Inverse Distance Weighting



Kriging



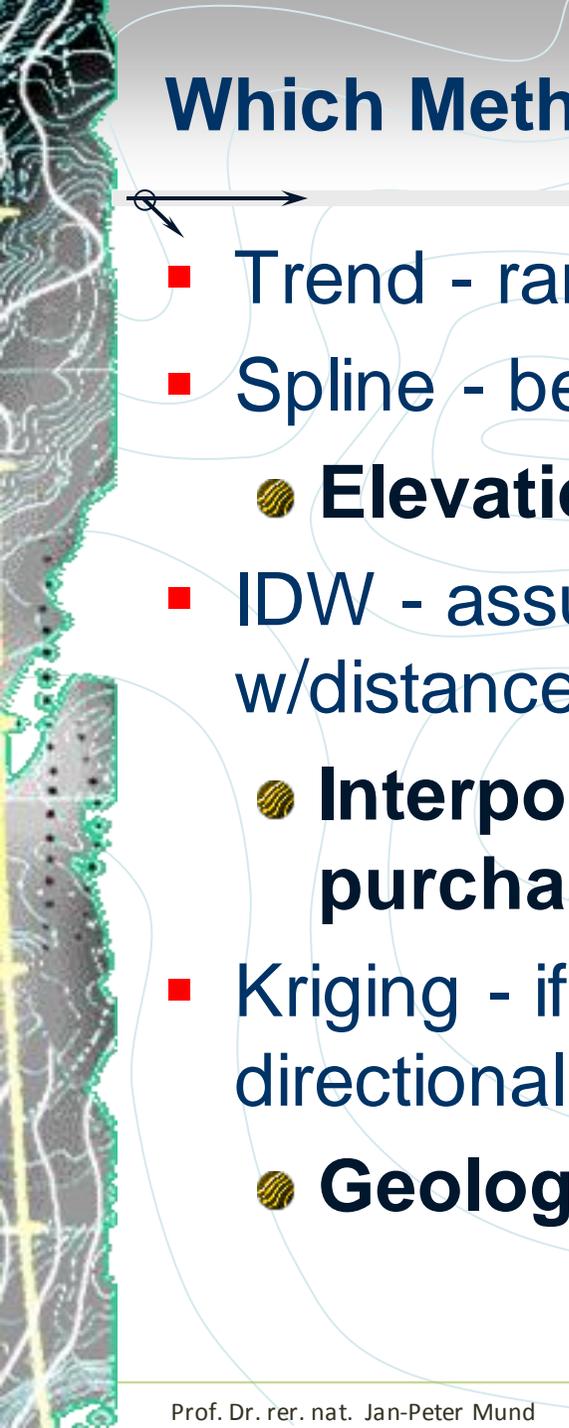
A Comparison of the Geostatistical Analyst methods

Method	Deterministic/ Stochastic	Output Surface Types	Computing Time/ Modeling Time ¹	Exact Interpolator	Advantages	Disadvantages	Assumptions ²
Inverse Distance Weighted	Deterministic	Prediction	Fast/Fast	Yes	Few parameter decisions	No assessment of prediction errors; produces "bulls eyes" around data locations	None
Global polynomial	Deterministic	Prediction	Fast/Fast	No	Few parameter decisions	No assessment of prediction errors; may be too smooth; edge points have large influence	None
Local polynomial	Deterministic	Prediction	Moderately Fast/Moderate	No	More parameter decisions	No assessment of prediction errors; may be too automatic	None
Radial basis functions	Deterministic	Prediction	Moderately Fast/Moderate	Yes	Flexible and automatic with some parameter decisions	No assessment of prediction errors; may be too automatic	None
Kriging	Stochastic	Prediction; Prediction Standard Errors; Probability; Quantile	Moderately Fast/Slower	Yes without measurement error; No with measurement error	Very flexible; allows assessment of spatial autocorrelation; can obtain prediction standard errors; many parameter decisions	Need to make many decisions on transformations, trends, models, parameters, and neighborhoods	Data comes from a stationary stochastic process, and some methods require that the data comes from a normal distribution
Cokriging	Stochastic	Prediction; Prediction Standard Errors; Probability; Quantile	Moderate/ Slowest	Yes without measurement error; No with measurement error	Very flexible; can use information in multiple datasets; allows assessment of spatial cross- correlation; many parameter decisions	Need to make many decisions on transformations, trends, models, parameters, and neighborhoods	Data comes from a stationary stochastic process, and some methods require that the data comes from a normal distribution

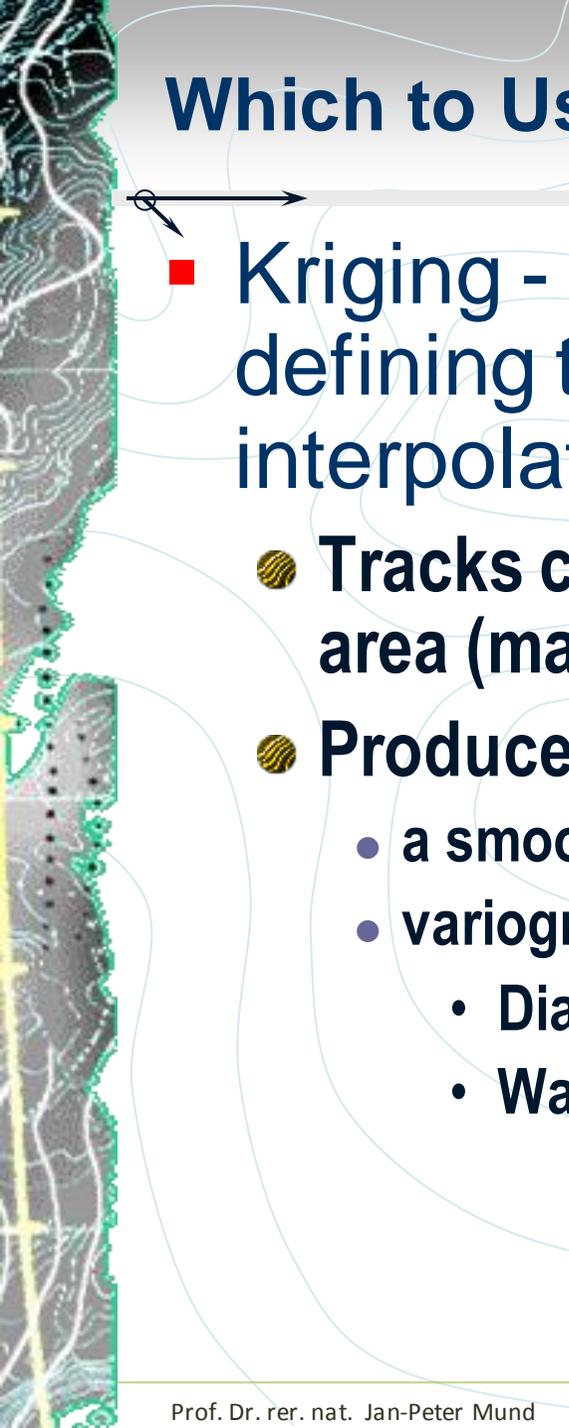
1. Computing time is computer-processing time to create a surface. Modeling time includes user-processing time to make decisions on model parameters and search neighborhoods.

2. We assume that all methods are predicting a smooth surface from noisy data.

Which Method to Use?

- 
- Trend - rarely goes through your original points
 - Spline - best for surfaces that are already smooth
 - **Elevations, water table heights, etc.**
 - IDW - assumes variable decreases in influence w/distance from sampled location
 - **Interpolating a surface of consumer purchasing power for a retail store**
 - Kriging - if you already know correlated distances or directional bias in data
 - **Geology, soil science**

Which to Use? *cont.*

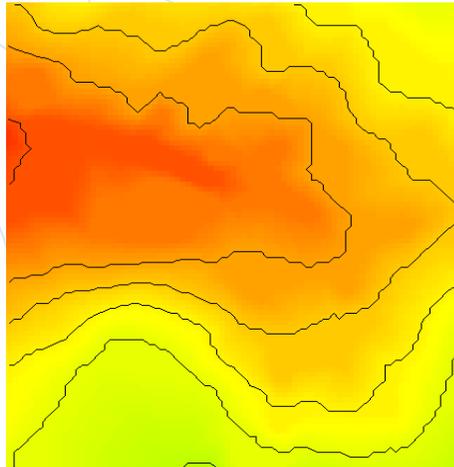
- 
- **Kriging** - Allows user greater flexibility in defining the model to be used in the interpolation
 - **Tracks changes in spatial dependence across study area (may not be linear)**
 - **Produces**
 - a smooth, interpolated surface
 - variogram (how well pixel value fits overall model)
 - Diagnostic tool to refine model
 - Want to get variances close as possible to zero

- 
- **ArcGIS with Geostatistical Analyst**
 - **ArcView 3.3**
 - **Surfer (Golden Software)**
 - **Surface II package (Kansas Geological Survey)**
 - **GEOEAS (EPA)**
 - **Spherekit (NCGIA, UCSB)**
 - **Matlab**

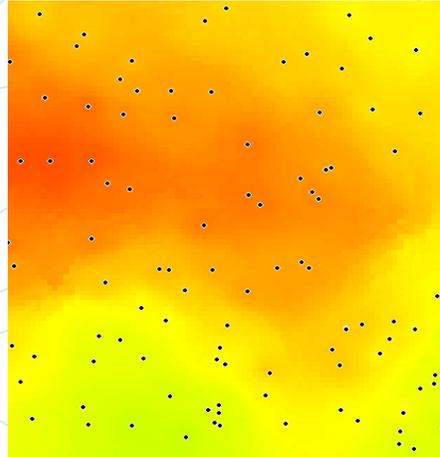
Effects of data uncertainty



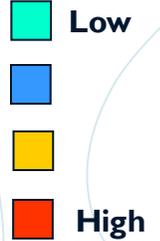
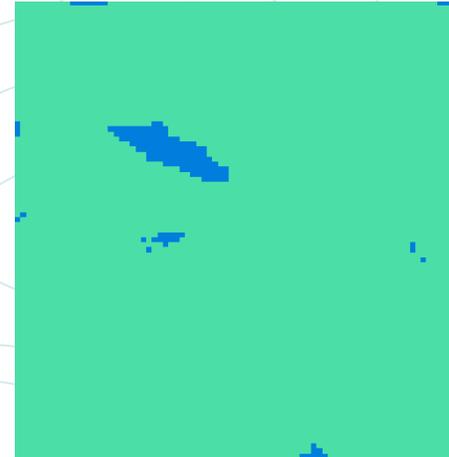
Original surface



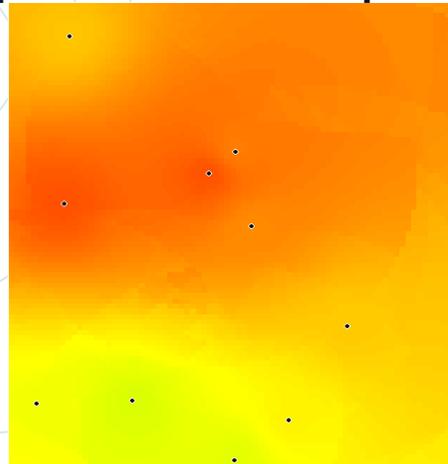
Interpolation based on 100 points



Error map



Interpolation based on 10 points



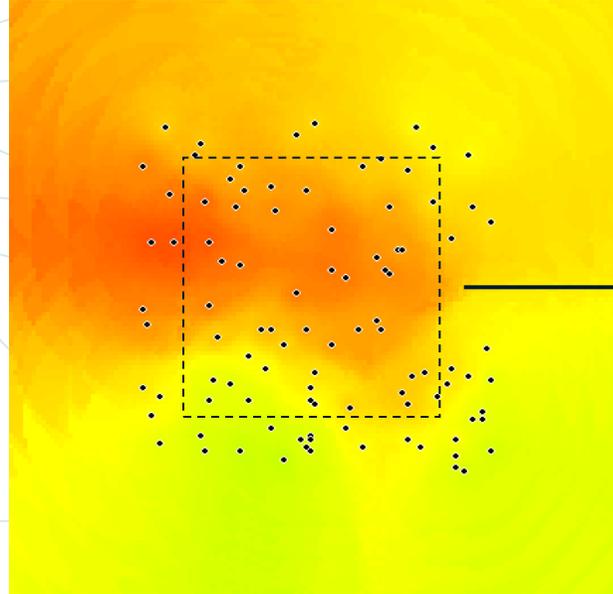
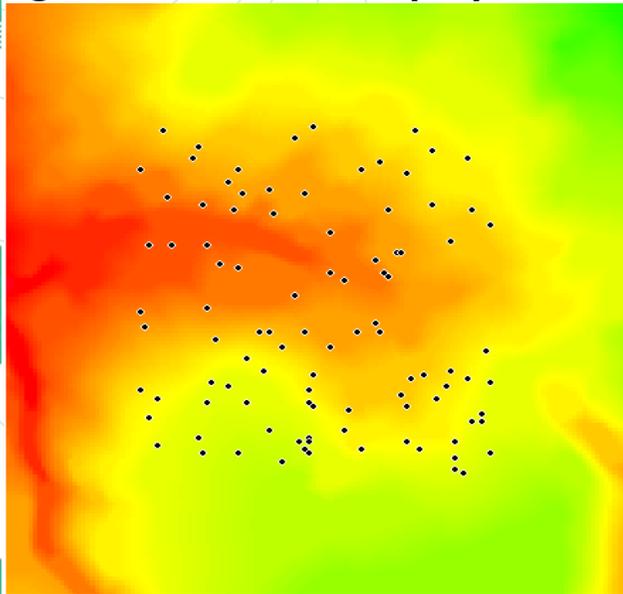
Error map



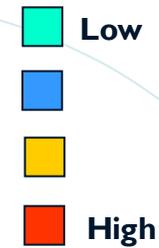
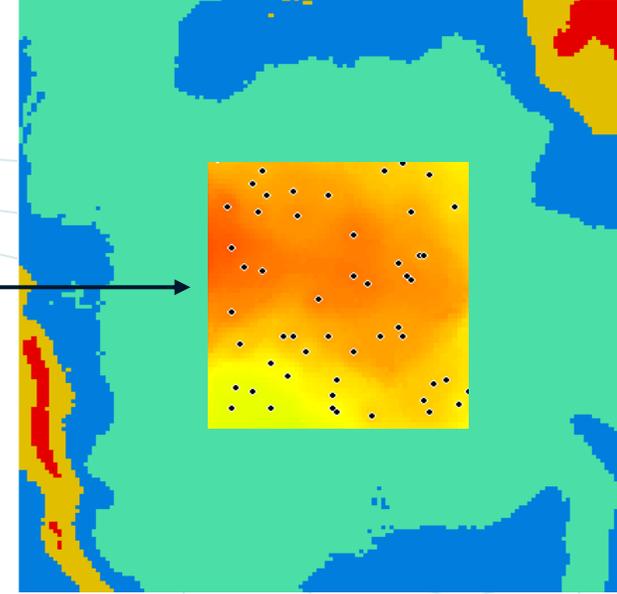
Edge effects



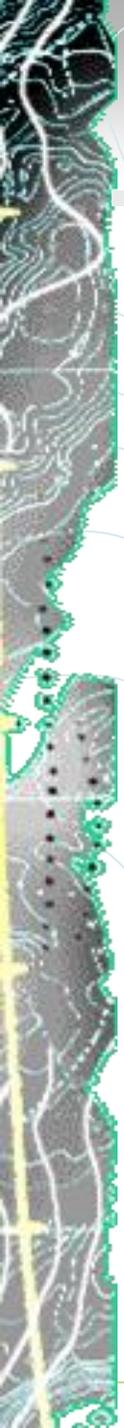
Original surface with sample points Interpolated surface



Error map and extract

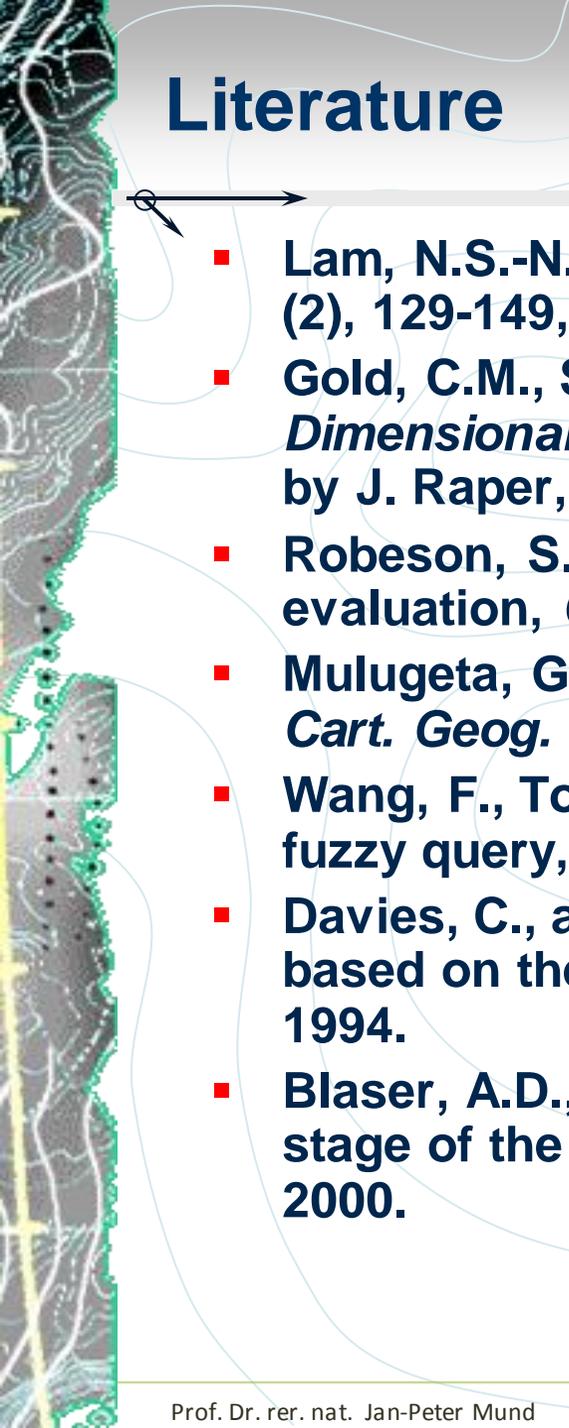


Conclusions

- 
- **Interpolation of environmental point data is important skill**
 - **Many methods classified by**
 - local/global, approximate/exact, gradual/abrupt and deterministic/stochastic
 - choice of method is crucial to success
 - **Error and uncertainty**
 - poor input data
 - poor choice/implementation of interpolation method

Conclusion

- 
- **Prediction maps were created using the training data set for all five interpolation techniques and projected values were estimated for the test data set.**
 - **Statistical parameters (error values) were evaluated and the prediction maps generated from these techniques were compared to the soil uranium concentration map.**
 - **It was inferred that any of the four (Ordinary Kriging, IDW, RBF and Local Polynomial) interpolation techniques can be used for predicting the radon concentrations for unmeasured zip codes.**
 - **Ordinary Kriging technique was chosen and the geometric means of radon concentrations were evaluated for unmeasured zip codes.**

- 
- **Lam, N.S.-N., Spatial interpolation methods: A review, *Am. Cartogr.*, 10 (2), 129-149, 1983.**
 - **Gold, C.M., Surface interpolation, spatial adjacency, and GIS, in *Three Dimensional Applications in Geographic Information Systems*, edited by J. Raper, pp. 21-35, Taylor and Francis, Ltd., London, 1989.**
 - **Robeson, S.M., Spherical methods for spatial interpolation: Review and evaluation, *Cartog. Geog. Inf. Sys.*, 24 (1), 3-20, 1997.**
 - **Mulugeta, G., The elusive nature of expertise in spatial interpolation, *Cart. Geog. Inf. Sys.*, 25 (1), 33-41, 1999.**
 - **Wang, F., Towards a natural language user interface: An approach of fuzzy query, *Int. J. Geog. Inf. Sys.*, 8 (2), 143-162, 1994.**
 - **Davies, C., and D. Medyckyj-Scott, GIS usability: Recommendations based on the user's view, *Int. J. Geographical Info. Sys.*, 8 (2), 175-189, 1994.**
 - **Blaser, A.D., M. Sester, and M.J. Egenhofer, Visualization in an early stage of the problem-solving process in GIS, *Comp. Geosci*, 26, 57-66, 2000.**



Blog

New in Geostatistical Analyst 10.1 : Areal Interpolation

by Eric Krause on [June 8, 2012](#)

[Share](#) 3 [Tweet](#) 5 [share](#) 37

For version 10.1, we've taken on a classic problem in GIS: how to reallocate data from one set of polygons to a different set of polygons. For example, demographers frequently collect data from various sources, so their data might be a mixture of census block groups, postal codes, and county boundaries. However, to perform an accurate analysis, they might need all of their data in the same administrative units.

While there are various methods for going from small polygons to large polygons (from census blocks to postal codes, for example), the benefit of areal interpolation is that it additionally provides a statistically accurate framework for going from large polygons to small polygons. By convention, the starting polygons are called the "source" polygons, and the ending polygons are called the "target" polygons. [Continue reading →](#)

Posted in [Analysis & Geoprocessing](#) | Tagged [Geostatistical Analyst](#), [interpolation](#), [polygon](#) | [3 Comments](#)

New in Geostatistical Analyst 10.1 : Empirical Bayesian Kriging

by Eric Krause on [June 8, 2012](#)

[Share](#) 0 [Tweet](#) 0 [share](#) 18

Those of you familiar with kriging interpolation know that it is not always the easiest technique to implement successfully. For a long time we've wanted to make a geoprocessing tool that can automate kriging, but the problem has always been in the complexity of calculating good default parameters. At 10.1, through a combination of subsetting and simulations, we have a solution to the problem with a method called empirical Bayesian kriging (EBK). The method is available in the Geostatistical Wizard and as a geoprocessing tool in the Geostatistical Analyst toolbox.

EBK works by building local models on subsets of the data, which are then combined together to create the final surface. Because the interpolation model is built automatically, the method requires very few parameters. There are also some optional parameters that give you some control over how locally the models will be built and how they will be combined together.

Why should I use EBK?

- Simplicity – To get accurate results, all you need to do is specify the field you want to interpolate. Other kriging methods require you to build the model step-by-step to be confident that the results are statistically accurate.
- Automation – Because EBK is available as a geoprocessing tool, you can use it in Model Builder and in Python scripts.
- Capture small-scale effects – Using local models allows EBK to capture small-scale effects that global kriging models may miss.

This post was contributed by Eric Krause, a product engineer on the analysis and geoprocessing team.

This Blog

[Sign in](#)

Technical Communities

[Developer\(256\)](#)

[ArcGIS Online\(342\)](#)

[Editing\(98\)](#)

[Mobile\(103\)](#)

[Mapping\(358\)](#)

[Analysis & Geoprocessing\(141\)](#)

[Services\(572\)](#)

[Web\(95\)](#)

[Geodata\(153\)](#)

[3D GIS\(65\)](#)

[Imagery\(144\)](#)

[Python\(14\)](#)

Industry Communities

[Community Maps\(38\)](#)

Thank you for your kind attention

